Journal of Experimental Psychology: General
© 2013 American Psychological Association
0096-3445/13/$12.00 DOI: 10.1037/a0032108

Is It Better to Select or to Receive? Learning via Active and Passive Hypothesis Testing

Douglas B. Markant and Todd M. Gureckis
New York University

People can test hypotheses through either *selection* or *reception.* In a selection task, the learner actively chooses observations to test his or her beliefs, whereas in reception tasks data are passively encountered. People routinely use both forms of testing in everyday life, but the critical psychological differences between selection and reception learning remain poorly understood. One hypothesis is that selection learning improves learning performance by enhancing generic cognitive processes related to motivation, attention, and engagement. Alternatively, we suggest that differences between these 2 learning modes derives from a *hypothesis-dependent sampling bias* that is introduced when a person collects data to test his or her own individual hypothesis. Drawing on influential models of sequential hypothesis-testing behavior, we show that such a bias (a) can lead to the collection of data that facilitates learning compared with reception learning and (b) can be more effective than observing the selections of another person. We then report a novel experiment based on a popular category learning paradigm that compares reception and selection learning. We additionally compare selection learners to a set of "yoked" participants who viewed the exact same sequence of observations under reception conditions. The results revealed systematic differences in performance that depended on the learner's role in collecting information and the abstract structure of the problem.

Keywords: hypothesis testing, self-directed learning, category learning, Bayesian modeling, hypothesis-dependent sampling bias

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

Hypothesis testing refers to the act of generating a set of alternative conceptions of the world, either explicitly or implicitly, and using empirical observations to verify or refine that set (Poletiek, 2011; Thomas, Dougherty, Sprenger, & Harbison, 2008). The ubiquity of this approach to inference is evidenced by its widespread study in many areas of psychology, including theories of category and concept learning (Bruner, Goodnow, & Austin, 1956; Nosofsky & Palmeri, 1998), perception (Gregory, 1970, 1974), social interaction (Snyder & Swann, 1978; Trope & Bassok, 1982; Trope & Liberman, 1996), logical reasoning (Wason, 1966, 1968), and word learning (Carey, 1978; Siskind, 1996; Xu & Tenenbaum, 2007b).

Bruner et al. (1956) presented a distinction between hypothesis testing through *selection* versus *reception.* During selection, a learner actively decides which observations to collect in order to test a hypothesis (Klayman & Ha, 1987; Skov & Sherman, 1986; Wason, 1960, 1966, 1968). For example, a doctor might decide to order a particular blood test based on hypotheses about a patient's illness. In contrast, hypothesis testing by *reception* is a passive mode of inference whereby the learner "must make sense of what happens to come along, to find the significant groupings in the flow of events to which he is exposed and over which he has only partial control" (Bruner et al., 1956, p. 126). Experimental tasks involving hypothesis testing usually fractionate along this distinction with fewer attempts to compare these two forms of learning under otherwise similar conditions. For example, early work in hypothesis testing focused mainly on selection tasks such as the rule discovery (or "2-4-6") task (Wason, 1960, 1966), whereas research on category and concept learning has typically relied on reception tasks in which examples are chosen by the experimenter (e.g., Nosofsky & Palmeri, 1998; Shepard, Hovland, & Jenkins, 1961). This segregation is affirmed by the fact that none of the leading models of category or concept learning have been designed to account for selection-based learning, despite the relevance of both modes of learning for concept acquisition (Hunt, 1965; Laughlin, 1972, 1975; Schwartz, 1966).

Selection and reception learning are not simply artifacts of laboratory research but capture a core distinction in the way that people refine their beliefs about the world. Sometimes we strategically design tests to evaluate our ideas, and other times we simply stumble upon the relevant data as part of our ongoing experience. Whereas Bruner et al. (1956) discussed this distinction in the context of concept learning, similar issues have been discussed in many related disciplines including education, philosophy, and machine learning. As one example, a long-debated idea in education is whether "active learning" (Bruner, 1961; Bruner, Jolly, & Sylva, 1976; Kolb, 1984; Montessori, 1912/1964; Papert, 1980; Piaget, 1930; Steffe & Gale, 1995) is more effective than

Douglas B. Markant and Todd M. Gureckis, Department of Psychology, New York University.

We thank Bob Rehder, Greg Murphy, Nathaniel Daw, Larry Maloney, Eric Taylor, and the NYUConCats group for helpful discussions in the development of this work. We also thank Erica Greenwald for help with the illustrations and figures.

Correspondence concerning this article should be addressed to Douglas B. Markant, Department of Psychology, New York University, 6 Washington Place, New York, NY 10003. E-mail: doug.markant@nyu.edu

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

more passive or guided forms of instruction (e.g., Klahr & Nigam, 2004). Understanding the differences between selection and reception learning may thus have a number of implications beyond cognitive psychology.

Despite decades of research on hypothesis testing, little is known about the psychological processes that distinguish learning via selection or reception. We propose that learning by selection introduces a *hypothesis-dependent sampling bias* wherein learners select new observations that test the specific hypothesis they currently have in mind. As a result, the pattern of data experienced during learning becomes tied to the particular sequence of hypotheses considered by the selection learner. This relatively simple idea has a number of interesting implications that we examine in our study. First, we show that selection learners are at an advantage in certain learning problems because they can optimize their training experience (e.g., by avoiding data they expect to be redundant under their current hypothesis). Second, because selection decisions depend on the learner's current hypothesis, the exact same sequence of training examples will be less useful to other learners. We assess this in our study using a "yoked" design wherein reception learners view the selections made by another learner.

The organization of the present article is as follows: We begin by reviewing prior work in philosophy, machine learning, and cognitive psychology that bear on the selection versus reception distinction. Next, we introduce a theoretical framework for understanding this distinction psychologically. We then present a novel empirical study in which learning via selection and reception is compared under otherwise equivalent conditions. The study allowed us to ask three key questions. First, which learning mode is faster or more effective? Second, does the advantage for a particular learning mode depend on the complexity of the concept being learned? Third, to what degree does any advantage for selection learning depend on the differences in data experienced or on more general factors related to learning (e.g., engagement or attention)? Finally, we describe a computational model that extends existing theories of concept learning in order to account for the effects of both reception and selection learning. The model makes concrete our theory of hypothesis-dependent sampling bias, and through a set of simulations, we show that this bias can explain the pattern of results in our empirical study.

The Informational Advantages of Learning by Selection

On first consideration, it may appear obvious that selection has the potential to accelerate the rate of learning compared with learning by reception. Instead of being limited by the passive flow of information, selection-based learners are free to gather data they judge to be useful or informative. At a minimum, this allows them to avoid redundant or irrelevant data and to focus on collecting information that is expected to result in further learning. For example, to learn which animals are included in the class RODENT, it may be more efficient to ask about uncommon, atypical animals ("porcupine") as opposed to frequently encountered, typical ones ("mouse") in order to find the boundary of the category. Learning via selection can thus be more efficient because it generates *more informative data* (assuming, of course, that the learner can gather data in a nonrandom, useful way).

The "informational advantage" of selection-based learning is a well-established principle of relevance to the design of machine learning systems. For example, teaching a system to automatically classify images or videos on the web often depends on training data that has been labeled by a human operator, but this annotation is costly and time-consuming. A more efficient system might be designed that only requests human annotations for items that are expected to be helpful for classifying other documents, as opposed to wasting time on items that can already be classified with relative confidence (Mackay, 1992; Settles, 2009). This basic idea underlies "active" machine learning techniques that have been shown to speed learning in a variety of problems, including sequential decision making (Schmidhuber, 1991), causal learning (Murphy, 2001; Tong & Koller, 2001), and categorization (Castro et al., 2009; Cohn, Atlas, & Ladner, 1992; Dasgupta, Kalai, & Monteleoni, 2005).

A similar principle is at play in scientific inquiry, a paradigmatic example of selection-based hypothesis testing. Scientists seek to verify theories about the way the world works by actively testing these theories in empirical studies. Work in the philosophy of science has considered which experiments a scientist should perform given a set of hypotheses. The two approaches that have attracted the most attention are seeking *confirmation* (performing tests or experiments that are expected to be consistent with the current hypothesis) or *falsification* (choosing tests which could potentially disconfirm the theory). Popper (1935/1959) influentially argued that falsification is the best way to accelerate the accumulation of knowledge. In other words, when done with the aim of falsifying a hypothesis, selection should lead to faster learning. This idea has a long history in the philosophy of science wherein "crucial experiments" (Bacon, 1620/1902) that decide between alternative theories should allow more effective discovery. A similar idea is contained within Platt's (1964) description of "strong inference,"which suggests that fields in which theories are systematically falsified are associated with more rapid progress.

Hypothesis testing research in cognitive psychology draws heavily from these ideas (see Poletiek, 2011, for a review). One of the major questions in the field has been whether people (particularly nonscientists) "intuitively" seek confirmation or falsification in everyday tasks, with widespread consensus that people are fairly biased toward confirmation of an existing hypothesis over falsification (Nickerson, 1998; Wason, 1960). However, in some environments, selecting confirmatory evidence (via the so-called positive test strategy; Klayman & Ha, 1987) has been shown to be an effective strategy for hypothesis testing (Austerweil & Griffiths, 2008; Navarro & Perfors, 2011). Moreover, people are sensitive to the relative usefulness of different observations when given a set of alternative hypotheses to evaluate (Nelson, McKenzie, Cottrell, & Sejnowski, 2010; Skov & Sherman, 1986), suggesting that people can select data in an efficient manner in some contexts.

One domain in which selection has been empirically shown to improve learning relative to reception is causal learning (Lagnado & Sloman, 2004, 2006; Sobel & Kushnir, 2006; Steyvers, Tenenbaum, Wagenmakers, & Blum, 2003). In this context, selecting data involves "intervening" on a variable by fixing its value and evaluating the effect of that intervention on other variables (Pearl, 2000). The distinction between these modes is critical because certain causal networks are indistinguishable through observation alone, and can only be learned through active intervention.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

Steyvers et al. (2003) tested whether people could acquire a causal relationship between three variables on the basis of either intervention or passive observation. Their results showed that people made interventions that were likely to be informative about the set of possible causal structures, and were more successful at learning the correct structure than people who learned by observation alone. However, this advantage may also derive from other processes involved in intervention-based learning, including the generation of temporal cues linking interventions with its effects on other variables (Lagnado & Sloman, 2004, 2006). It is unclear whether the processes that contribute to these performance differences also play a role in problems that do not involve learning about causal relations.

Despite the theoretical rationale and empirical support for an informational advantage for selection-based learning, it may not always be the best strategy for learners to adopt (Enkvist, Newell, Juslin, & Olsson, 2006; Schwartz, 1966). In particular, it may be less effective than reception when learning more difficult or complex concepts. For example, Schwartz (1966) found that although selection was most efficient for learning simple rules (i.e., a concept based on a single binary feature), disjunctive rules were learned faster through reception of randomly chosen examples. As yet, however, there have been few systematic investigations into how the benefits of selection might depend on the structure or complexity of the target concept.

Measuring the Impact of Selection Decisions Through "Yoked" Experiments

Aside from the potential advantages of selecting better data, a separate line of research has questioned whether there are general cognitive benefits from learning by selection. One way to test this is through experiments in which a "yoked" reception participant learns from the same data that is gathered by a selection learner. By holding the sequence and content of training data constant across the pair of individuals, this design aims to isolate the effect of making selection decisions on learning (Gureckis & Markant, 2012). One might expect that the yoked participant would have an advantage because they are spared the additional demand of selecting data, and in fact this was the conclusion of one of the earliest examples of this design (Huttenlocher, 1962). However, other studies of this kind have found that selection learners are more successful than their yoked counterparts, despite the fact that each pair experiences an identical set of data (Hunt, 1965; Lagnado & Sloman, 2004; Schwartz, 1966; Sobel & Kushnir, 2006; Steyvers et al., 2003).

What can account for the advantage for selection-based learning when the training experience is held constant? One common explanation is that selection and reception are distinct cognitive "modes" that differ with respect to processes such as attention, memory, or motivation. For example, selection learners may have improved memory for individual training items (Voss, Gonsalves, Federmeier, Tranel, & Cohen, 2011) or may attend to a different subset of stimulus features from a yoked learner (Smalley, 1974). Alternatively, making choices during learning may entail deeper processing of the task (Kuhn & Ho, 1980; Sobel & Kushnir, 2006) or may simply be inherently rewarding or engaging. In short, learning by selection may lead to an enhancement of some cognitive process or the recruitment of additional processes that facilitate learning (Chi, 2009; Elio & Lin, 1994).

Alternatively, selection learners may benefit from their ability to collect information that specifically tests their individual hypothesis (Hunt, 1965). The resulting *hypothesis-dependent sampling bias* may lead to a training experience that is uniquely optimized to refine their existing knowledge, whereas the same experience may be poorly matched to the mental state of a yoked partner (we use the term *bias* in the sense that the selection of data depends on the learner's current beliefs, rather than to imply that those decisions are made in error). Differences between a pair of learners may emerge simply because each individual is likely to have different hypotheses in mind at any point in time, but the sequence of data is more closely linked to the hypotheses of the selection learner.

For example, imagine a social learning situation in which two children are learning to skip stones across the surface of a lake. Each child may have different initial ideas about what makes a "good" skipping stone: One child may believe that the flatness of the stone predicts whether it will skip, whereas the other believes that the weight of the stone is the feature that matters. If only one child picks new instances to test, however, the sequence of observations may be most informative about their specific hypothesis (e.g., the first child might try stones that vary in flatness but are all about the same weight). The "yoked" child who watches the other's selections may still learn from those observations, but he or she is likely to learn more slowly than if he or she had been able to test his or her own hypothesis in the first place (we provide a quantitative demonstration of this idea in the next section). Importantly, this divergence can arise without assuming any differences in terms of the learning process or overall motivation. In addition, the magnitude of the divergence will depend on *how biased* the selection learner's observations are. If a selection learner chooses new data completely at random, then that data are as likely to be useful to him or her as it is to a yoked observer. As the learner makes selections that are more closely tailored to his or her current hypothesis, however, his or her advantage over the yoked partner will increase.

Accounting for the benefits of learning by selection, particularly when the actual content of the training experience is matched, requires considering both of these explanations. Whereas previous work has largely focused on a general "engagement" hypothesis—that selection and reception are distinct cognitive modes of hypothesis testing—in the present article we show that a simpler, more computationally grounded theory based on a hypothesis-dependent sampling bias may also contribute to differences between these two learning modes. In the next section, we provide a concrete illustration of this principle before turning to an experimental test of this idea.

Implications of a Hypothesis-Dependent Sampling Bias for Learning

The central idea behind a hypothesis-dependent sampling bias—that actively testing one's own hypothesis can be different from observing someone else test his or hers—is psychologically plausible and intuitive (see Schober & Clark, 1989, for a similar idea in the context of participating in vs. overhearing a conversation).

However, rather than rely on intuition, a goal of the present article was to articulate a theory of the difference between reception and selection learning in terms of computational principles. In this section, we describe how a hypothesis-dependent sampling bias can lead to an advantage for selection learning as compared with passive reception of randomly generated training data, as well as "yoked" reception of a sequence of data chosen by a different selection learner. The details of this preliminary simulation help to set up the core aims of our later experiment.

Returning to the example we introduced in the previous section, imagine learning which stones will skip across the surface of a lake. For simplicity, assume that stones on this particular beach are identical except for their relative flatness (see Figure 1, top). In this situation, a single hypothesis is an idea about which level of flatness correctly divides the stones into two categories: those that will skip and those that will sink (see Figure 1, middle). In addition to the two illustrated, a wide variety of hypotheses are possible, each corresponding to a different threshold along the dimension of flatness. The goal of learning is to determine which criterion along the flatness dimension corresponds to the true state of the world. In a selection task, the learner also has to decide which item to learn about next. The blue line in Figure 1 (bottom) indicates a range of possible selections the learner might make on the next learning trial.

Although simplified, this scenario incorporates all of the relevant features of more general hypothesis-testing situations: There is a range of plausible hypotheses, each hypothesis makes different predictions about observable events in the world, and empirical observations are available that could help determine which hypothesis is correct (Poletiek, 2011). We now consider the implications of learning through selection or reception given this basic process. The learner could (a) pick completely at random (e.g., picking up a stone without looking at its flatness first), (b) pick new examples through self-directed selection, or (c) receive examples through "yoked" observation of another learner's selections.

Hypothesis Testing via Bayesian Learning

It is instructive to consider the implications of each of these three data-gathering processes for a "weak" Bayesian learner that represents and updates the full set of alternative hypotheses while learning.[1] According to Bayes' rule, the posterior belief about each hypothesis h can be written as:

$$p(h|D) \propto p(D|h)p(h), \tag{1}$$

where $p(h)$ is the prior belief in each hypothesis, $p(D|h)$ is the likelihood of the observed data D given the hypothesis h, and $p(h|D)$ is the subjective belief about hypothesis h after observing the data. The magnitude of the value $p(h|D)$ for each hypothesis reflects the relative strength of belief that this hypothesis is correct. Assuming deterministic hypotheses, weak sampling implies that observations consistent with a hypothesis have equal likelihood (e.g., $p(D|h) = 1$), whereas for any inconsistent observation, $p(D|h) = 0$. Following each observation of the skipping ability of a stone, any hypotheses that are inconsistent with that observation are falsified and will have $p(h|D) = 0$. Figure 2 illustrates this process for a subset of possible hypotheses, showing that on each trial, hypotheses are removed that are inconsistent with the data experienced so far (note that the full hypothesis space would include many hypotheses not shown, including those in which flat stones are predicted to sink rather than skip, but the basic process is the same).

Given this model of learning, a selection learner should choose new observations that take into account the plausibility of alternative hypotheses. It is uninformative to test an item that all remaining hypotheses agree will either skip or sink. Instead, the learner should focus on items that will further refine the set of hypotheses, which in this case entails testing an item that falls somewhere between the closest positive and negative examples. This is shown in Figure 2 where the set of potential observations on each trial (shown by the blue line) is limited to those that will be able to discriminate between remaining hypotheses. Notice that as new observations are encountered, certain hypotheses are falsified while others remain viable.

As in the examples cited earlier, this kind of selection process leads to more efficient learning than random generation of data, for the simple reason that the "pool" of possible observations is limited to those that will be informative (e.g., if there are only two possible hypotheses left, the only informative data lies in the interval between them, and it would be inefficient to select anything else). Importantly, an identical outcome would be found for a second learner who is yoked to the data generated by a selection learner, because both begin with the full set of hypotheses, and the same subset would be disconfirmed with each new piece of data. Thus, according to a weak Bayesian model, learning by selection and *yoked* reception are equally superior to reception of randomly sampled instances. If all learners are assumed to represent the full range of plausible hypotheses, they also stand to benefit equally from the data that are selected.

Sequential Hypothesis Testing and the Hypothesis-Dependent Sampling Bias

We can contrast the qualitative predictions of this Bayesian model against an alternative model that is more limited in its capacity to consider alternative hypotheses at each point in time. Specifically, we consider a model based on sequential hypothesis testing that is representative of a large body of work on rule learning (Gregg & Simon, 1967; Millward & Spoehr, 1973; Nosofsky & Palmeri, 1998; Trabasso & Bower, 1968). The model makes two key assumptions that differ from the Bayesian model.

First, rather than consider all possible category boundaries between "skip" and "sink," it assumes that the learner considers only a single hypothesis on each trial. Following each observation, the learner evaluates whether the current hypothesis has been disconfirmed and, if so, generates a new one that is consistent with the new data. If the observation is consistent with the currently entertained hypothesis, then that hypothesis will be maintained for the next trial. This type of "win-stay, lose-shift" process is common to many existing theories of rule learning and hypothesis testing

[1] Following Tenenbaum (1999), we refer to this model of learning as *weak* because it is a straightforward application of Bayes's rule and does not incorporate assumptions about how the data were sampled (Mitchell, 1997; Shafto et al., 2012; Tenenbaum, 1999; Tenenbaum & Griffiths, 2001) The purpose of this example is explanatory and we later explore alternative Bayesian approaches in more detail.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

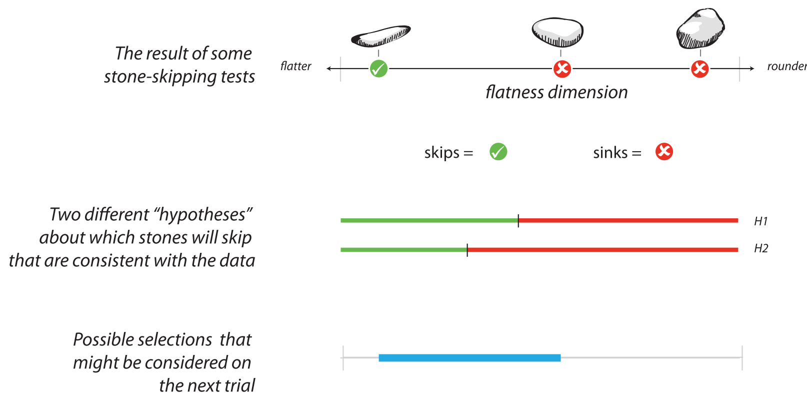

Figure 1. Which stones will skip and which will sink? In this example, the learner's goal is to infer how the property of flatness is related to a stone skipping. Individual stones can be thrown and will either skip (green check mark) or sink (red cross). The top part of the figure shows three example stones that were thrown across the lake, and the two rounder examples sank while the flatter example skipped. The middle part of the figure shows two possible hypotheses consistent with the data. Each hypothesis divides the dimension of "flatness" into stones that should skip (green) and those that should sink (red). In a selection environment, the learner also has to decide which stone to test next. The range of examples under consideration is illustrated by the blue line in the bottom panel. Notice that, in this example, the blue line extends between the two closest positive and negative examples.

(Gregg & Simon, 1967; Nosofsky & Palmeri, 1998; Trabasso & Bower, 1968). For example, RULEX considers a single rule at a time (starting with simpler rules) and generates more complex alternatives as it encounters data that are inconsistent with the rule currently held in memory (Nosofsky & Palmeri, 1998).

The second assumption pertains to how data are gathered during selection learning. We assume that there is a hypothesis-dependent sampling bias such that the data selected during self-directed learning are determined or constrained by the hypothesis the learner is currently considering. In particular, we assume that learners prefer to select items that are expected to lead to additional learning. Such items will generally occur near the learner's current estimate of the category boundary because this is where there may be greater uncertainty about how to classify items and where the learner is most likely to make mistakes.[2] In contrast, items that are far from the boundary are likely to be classified quickly and with high confidence (e.g., Ashby, Boynton, & Lee, 1994). The bias to select items at the category boundary can presumably vary in strength between individuals. For example, a strong hypothesis-dependent sampling bias would lead to observations that are tightly clustered around the learner's current estimate of the category boundary, whereas a weak bias would result in a wider distribution of observations (see Figure 3, top).

These two assumptions together lead to a divergence between selection and yoked learners' speed of acquisition. Figure 4 shows the result of simulating this model under the three data selection strategies described earlier (selection, yoked reception, and random reception; refer to the Appendix for full details). In the selection simulation, the model chose new observations according to the principle just described. In the random reception simulation, the model was presented new examples sampled from a uniform distribution over the entire range of "flatness" values. In the yoked reception condition, the model was presented with the same observations made by the selection model (the only other difference between the two conditions being their initial hypotheses). In the left panel, there is a faster drop in error for the selection model compared with the reception model learning via randomly selected examples. In addition, there is a gap in learning performance between the selection learners and the yoked learners even though both groups view the same exact sequence of data.

This gap between conditions arises without assuming any other differences between learners aside from their initial hypotheses and their ability to select data to test their own hypothesis. This result qualitatively differs from the Bayesian model described in the previous section, which predicts no difference between selection and yoked learners.[3] However, note that the extent of the advantage for selection is dependent on the strength of the hypothesis-dependent sampling bias. When the selection learner's choices are only weakly influenced by their current hypothesis, there is no difference between their performance and that of their yoked partner (see Figure 4, right panel).

This divergence in performance between selection learners and their yoked partners occurs because the sequence of data generated is more useful for the selection learner than the yoked reception learner. Figure 3 illustrates this asymmetry when the selection learner begins a task with a hypothesis that is too low (left column)

[2] Increased uncertainty at the boundary of the category could arise from a number of sources, including perceptual noise that makes classification decisions more difficult close to the boundary. Alternatively, the learner may have a more graded representation of the two categories (e.g., they may use dissimilarity from a category prototype to predict category membership). Increased uncertainty at the boundary between two categories is a common prediction of almost all categorization models, including exemplar, prototype, and decision-bound models.

[3] Note that the two models as described so far differ in the number of past observations that are stored (with a "full memory" in the Bayesian model and just a single item stored in the hypothesis-testing model). However, a difference between selection and yoked reception would not be predicted by the Bayesian model even with a limited memory, because the posterior distribution is updated in the same way for both kinds of learners, based on the same (albeit limited) set of data.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

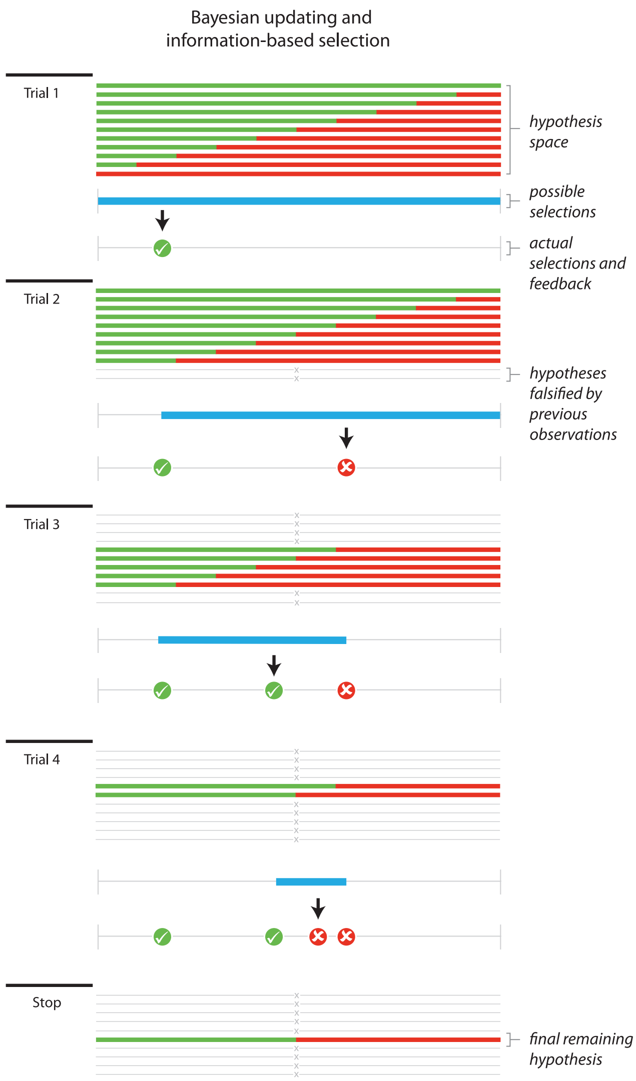

Figure 2. A Bayesian learner represents the full space of possible hypotheses (here, a discrete tiling of the space is shown for illustrative purposes as a "stack"). Each hypothesis divides the observation space into two regions (green for stones that skip and red for stones that sink). On each trial, a new observation is selected if it will reduce uncertainty about remaining hypotheses (shown by the blue interval labeled *possible selections*). This is because there is more disagreement between the remaining hypotheses in this region of the observation space. Under this model of learning, selection will be advantageous compared with reception of randomly generated data because it avoids redundant data, but no difference is expected in a *yoked* reception group without introducing additional assumptions about the differences between these two groups.

while their yoked partner begins the task with a hypothesis that is too high (right column). When the selection learner observes an unexpected outcome to the right of their hypothesized category boundary (i.e., a relatively round stone that skips), their hypothesis is disconfirmed, and they will generate a new one (with a good chance of generating a hypothesis closer to the true boundary). In contrast, the same observation is consistent with the yoked learner's initial hypothesis and will not lead to any adjustment. The consequence of this process is that, in general, the probability of falsifying the current hypothesis is greater for the selection learner, and they will have a faster rate of learning over a number of trials (as verified in Figure 4; see also the Appendix for a further demonstration).

Overview of the Present Study

The preceding analysis provides a mechanistic explanation for differences between selection and reception learning. We showed how specific performance differences may arise due to the interaction of a hypothesis-dependent sampling bias along with plausible assumptions about learning shared by many models of hypothesis testing and rule learning. By explicitly linking the state of the learner to the pattern of data experienced, this process can account for the benefits of selection learning as compared with randomly generated data, while also explaining why that advantage may not transfer to a yoked participant who views the same sequence of observations.

The goal of the present study was to empirically evaluate aspects of this theory by comparing learning via reception and selection in a single task. Our experiment design was based on a popular perceptual category learning paradigm (Ashby, Alfonso-Reese, Turken, & Waldron, 1998; Ashby, Maddox, & Bohil, 2002). Category learning is an ideal target for investigation because it is a well-studied task with broad implications for theories of concept acquisition. In addition, although there is general agreement in the field that many forms of category learning involve an element of hypothesis testing (Ashby et al., 1998; Erickson & Kruschke, 1998; Nosofsky & Palmeri, 1998; Trabasso & Bower, 1968), it has primarily been studied using reception-based training, making the comparison in our experiment relatively novel. Note, however, that the general principles at play (the distinction between selection and reception learning) likely transcend the particulars of the experimental task we selected and apply to other paradigms that involve an element of hypothesis testing such as intervention-based causal learning.

As in our simulation, we compared learning performance in (a) a standard reception learning condition, (b) a selection condition in which learners actively choose category exemplars to learn about on each trial, and (c) a yoked reception condition in which learners see the same exemplars chosen by a person in the selection condition. In addition to this key contrast, we were also interested in how the differences between these conditions might interact with the complexity of the rule the learner is attempting to discover.

Our experimental results are followed by a more detailed model-based analysis based on the framework just presented. Most importantly, we show how the basic principles of the hypothesis-dependent sampling bias can explain differences in performance between selection, reception, and yoked reception conditions *without* assuming that they involve distinct cognitive "modes" or differing sets of parameters as has frequently been suggested in prior work. This contribution is important because many existing theories of category learning (including the weak Bayesian model described above) predict that "learning-by-doing" (i.e., selection) and "learning-by-observing" (i.e., reception) should result in identical patterns of behavior.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

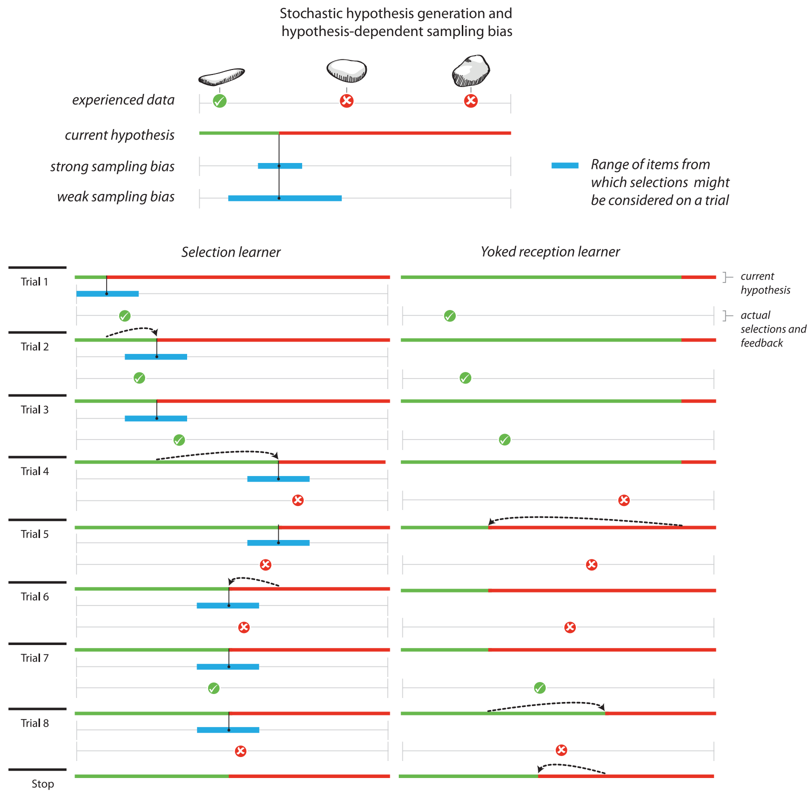

Figure 3. Top: Unlike the weak Bayesian learner, in sequential hypothesis testing, the learner considers just a single hypothesis about the boundary between stones that skip and stones that sink. During selection learning, new observations are biased by the location of the learner's current hypothesis (blue line), with the strength of the bias determining how close new items are chosen to the hypothesized boundary. Bottom: Selection and yoked learners begin with different initial hypotheses, but only the selection learner can test his or her hypothesis (choosing an item from the blue interval). If an observation leads to falsification of the current hypothesis (e.g., a stone that falls to the right side of the boundary that also skips, as on the first trial for the selection learner), a new hypothesis is randomly generated that is consistent with that observation (dashed arrows). The same observations generated by the selection learner are observed by the yoked reception learner, who learns via the same process. The observations generated are more likely to lead to falsification of the selection learner's hypothesis on each trial than the yoked reception learner.

Experiment

A relatively simple and well-studied perceptual category learning task involving multidimensional, continuous-valued stimuli (Ashby et al., 1998, 2002) was used in the experiment. In the task, participants learned to classify perceptual stimuli into one of two groups. Two types of abstract category structures were used: (a) a *rule-based* (RB) structure, in which the optimal classification rule is a criterion along a single dimension and (b) an *information-integration* (II) structure, in which the optimal classification rule is a linear combination of the values along two dimensions (see Figure 5A). We anticipated that participants' overall ability would vary between the RB and II learning tasks, as previous research has suggested that these two category structures may be learned in different ways (Ashby et al., 1998).

In addition to the two category structures, participants in the experiment were further divided into four training conditions. In the *selection* (S) condition, participants "designed" stimuli in order to reveal their category membership. In the *random-reception* (R) condition, participants observed training stimuli that were randomly generated from two bivariate normal distributions (i.e., a standard training procedure for these types of tasks).

In the final two training conditions, participants were "yoked" to the sequence of observations made by participants in the selection condition, but learned through passive observation. In the *naïve yoked reception* (Y1) condition, participants were not given any information about the source of their training data (i.e., they were in the same informational state as the random-reception participants). In the *aware yoked reception* (Y2) condition, participants were told that their data had been selected by a previous participant in the task with the same learning objective but who had learned through selection. The purpose of this last condition was to evaluate whether knowledge about how the observations had been selected would influence participants' inferences (e.g., Shafto, Goodman, & Frank, 2012).

We had two primary goals with this design. First, we were interested in whether selection learners would acquire the target

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

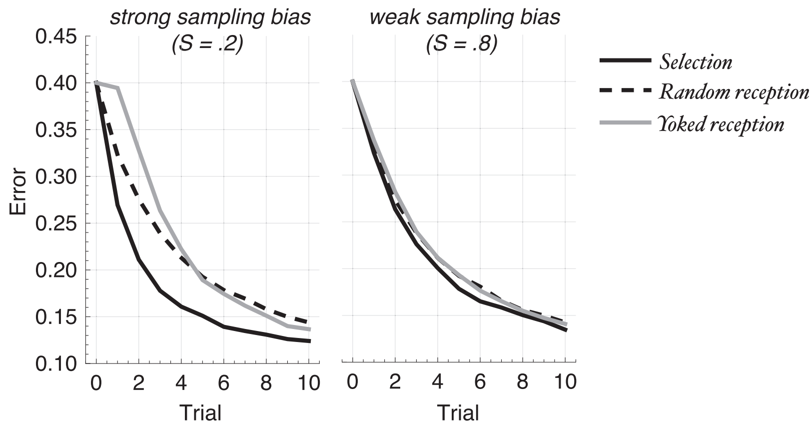

Figure 4. The effect of training condition on error rate for the sequential hypothesis testing model. In the left panel, selection learners achieve a lower error rate using fewer trials, evidence of faster learning. In addition, there is a large difference in performance between the "yoked" and selection model. This is evidence of a learning advantage from choosing your own observations to test. The right panel shows that this advantage depends on the hypothesis-dependent sampling bias. If the selection learner chooses more randomly (as compared to the simulation shown in the left panel), then the divergence between the conditions is expected to be reduced or absent.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

concept more quickly than through passive, reception-based learning for each kind of category structure. RB categories are thought to be learned by reasoning about verbal or explicit hypotheses, whereas II categories preclude a simple verbal description and are thought to be learned via implicit or procedural learning. Critically, it is often argued that learners initially try out simple, unidimensional rules and only abandon that strategy following extensive trial-and-error training. One source of evidence for this bias is typical behavior in the II task, in which early responses are best fit by suboptimal, unidimensional decision boundaries, and learning of the correct II boundary emerges slowly over the course of training (Maddox & Ashby, 2004). Given the relatively small amount of training data and the use of observational training, our II task likely involves an early stage of learning in which participants primarily rely on simple rules (Ashby et al., 2002). We hypothesized that selection learning may be most effective when participants are considering simple rules, and as a result would lead to an advantage in the RB case because the category structure aligns with that default strategy (Ashby, Queller, & Berretty, 1999; Kruschke, 1993). The comparison of selection learners with the random-reception group allowed us to measure any advantage for selection relative to the typical training experience in this kind of task.

Second, the inclusion of the yoked reception groups allowed us to separately evaluate the impact of selecting samples from the informational content of those selections. Previous research suggests that active or intervention-driven learning may lead to advantages over yoked learning (Lagnado and Sloman, 2004; Sobel & Kushnir, 2006; Steyvers et al., 2003), but it is unclear whether these results would generalize to both the RB and II category structures tested here. In addition, a factor that has not been examined in previous work is whether awareness of the yoking procedure is sufficient to overcome this disadvantage, which we test with our comparison between "naïve" and "aware" yoked groups. This large factorial design allowed us to simultaneously assess multiple influences on learning.

Method

Participants. Two hundred forty undergraduates at New York University participated in the study. The experiment was run on standard Macintosh computers in a single 40-min session. Each participant was assigned to either the rule-based (RB) or information-integration (II) task, and to one of four training conditions: selection (S), random reception (R), naïve yoked reception (Y1), or aware yoked reception (Y2).

Stimuli and materials. Stimuli were defined by a two-dimensional continuous-valued feature space, where one dimension corresponded to the size (radius) of a circle and the second dimension corresponded to the angle of a central diameter (see example in Figure 5B). Stimuli of this type have been used in many studies of perceptual classification (e.g., Garner & Felfoldy, 1970; Nosofsky, 1989; Shepard, 1964), and previous work has established that these two dimensions are, for most participants, separable and independent (Nosofsky, 1989). Stimuli could be assigned a value on each dimension within the range [1,600]. These values were converted for display such that there was a limited range of possible orientations and sizes. The orientation of the stimulus could vary over 150°, ensuring that a full rotation of the stimulus was not possible. The minimum radius and orientation were randomized so that the optimal decision boundary corresponded to a unique location in perceptual space for each participant.

One hundred twenty-eight training stimuli were created for the R training condition using samples from two bivariate Gaussian distributions (see Figure 5A), with mean and covariance parameters slightly modified from Ashby et al. (2002). For classification trials, a uniform grid of 256 unique test items was generated over the feature space for use in all conditions. For

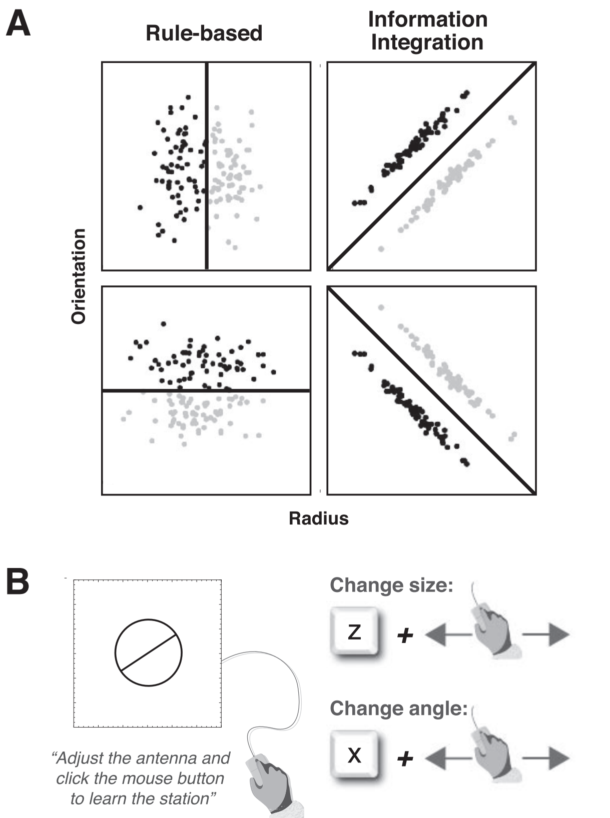

Figure 5. A: Examples of the rule-based and information integration category distributions. The space of stimuli is defined by two dimensions (orientation and radius, see panel B for an example stimulus). Each point in the space corresponds to a particular stimulus with a given value along each dimension. The clouds of points illustrate an example distribution of training stimuli shown to participants in the random reception condition, with the optimal decision boundary shown by the solid line. Participants in the other conditions received feedback consistent with this optimal boundary even though their training distribution differed from the training distribution plotted here. B: A depiction of a stimulus (left) and the interface used in the self-directed learning condition.

each test block, eight stimuli were randomly chosen (without replacement) from each quadrant of the stimulus space to avoid random biases in the test distribution, for a total of 32 items in each block. The order of individual test items within each block and the order of the eight test blocks were both randomized for each participant.

Procedure. Participants were told that the stimuli in the experiment were "loop antennas" for televisions and that each antenna received one of two channels (CH1 or CH2). They were told that the channel received by any antenna depended in some way on the two dimensions described above, and the participant's goal was to learn the difference between the two types of items. Participants were instructed that the antennas were sometimes "noisy" and could pick up the wrong channel "on occasion" and that it would be beneficial to integrate over a number of trials during learning (i.e., that they should learn what channel was "most often" received by a particular type of antenna). In this experiment, however, the feedback associated with each item was deterministic. The experiment consisted of eight blocks, with each block divided into a set of 16 training trials followed by 32 test trials.

Training phase: All conditions. The overall design of the training phase roughly matched the "observational learning" procedure used by Ashby et al. (2002). In that study, participants viewed a stimulus for a short, fixed duration followed by the corresponding category label of the stimulus for a fixed duration (the "no response/after" condition). Critically, participants were not asked to make an explicit prediction, and corrective feedback was never provided. The observational learning procedure is ideal for studying self-directed learning because we wanted to limit the conflicting demands of sampling informative items and sampling items that would result in "correct" feedback under a supervised procedure.

Training: S condition. On each training trial, the participant "designed" a TV antenna and learned about its category membership. Each trial began with the presentation of a randomly generated stimulus in the center of the screen. The participant could then alter its size and orientation by moving the mouse from left to right while holding down either the *Z* or *X* key (see Figure 5B). The direction of motion and mapping of keys to features were randomized across participants. Only one dimension could be changed at a time, but participants could make any number of changes and use as much time as needed. When the stimulus was the desired size and orientation, participants pressed the mouse button to reveal the category label, which appeared above the stimulus and was visible for 1,500 ms. Querying the category label was not permitted until the participant had made a change to the initial stimulus. Trial duration was recorded starting with the initial presentation of the stimulus until the end of the trial.

Training: R condition. In the R condition, participants were unable to interact with the stimuli in any manner. Instead, in each trial they were presented with a stimulus generated from the category distributions described in Table 1. On each trial, a fixation cross was presented, followed by the stimulus (for 250 ms), followed by the category label and stimulus together. Out of concern that in this passive, observational condition participants might not pay attention during the learning phase (relative to the S participants who interacted with the display), the participant was required to press a key corresponding to the displayed category in

Table 1
Category Distribution Parameters Used in the Experiment for the Random-Reception Condition

Condition	μ_x	μ_y	σ_x^2	σ_y^2	cov_{xy}
Rule based (RB)					
1. Category A	220	300	2,000	9,000	0
Category B	380	300	2,000	9,000	0
2. Category A	300	220	9,000	2,000	0
Category B	300	380	9,000	2,000	0
Information integration (II)					
1. Category A	250	350	4,538	4,538	4,463
Category B	350	250	4,538	4,538	4,463
2. Category A	250	250	4,538	4,538	−4,463
Category B	350	350	4,538	4,538	−4,463

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

order to end the trial. The stimulus and label remained visible on the screen until the verification response was registered.[4]

Training: Y1 condition. The purpose of the yoked conditions was to mimic the reception training experience, but using a sequence of observations that were chosen by a participant in the S condition. Each yoked participant was assigned to a participant in the S condition who had already completed the study. Training samples from the selection participant were used as the set of training items for the yoked participant, and were presented in the same order as they had been generated by the selection participant. All other aspects of the procedure were identical to the R condition. In the Y1 condition, participants were given no information about the source of the items they experienced during training.

Training: Y2 condition. In the Y2 condition, we manipulated the instructions given to participants to describe the task. At the start of the instructions, participants were told that they would be randomly assigned to one of two roles for the experiment: either a "Designer" or an "Apprentice" (a ploy to increase the belief that there really were two possible roles in the experiment). They were then all told that they had been assigned the role of Apprentice and that they would learn about antennas that had previously been selected by a Designer (the selection participant). Participants were also given a small number of selection training trials so that they were familiar with how the antennas were designed in the selection condition. These practice trials were followed by the instruction:

> You will learn about the exact same antennas that were created by the Designer you've been partnered with, using the design process you just practiced. You will see their designs in the exact same order as they were tested by the Designer. Keep in mind that the Designer wasn't told that their designs would be used to teach an Apprentice, but they were trying to learn the same thing as you.

All other aspects of the procedure were identical to the other reception conditions.

Test: All conditions. Each set of 16 training trials was followed by 32 test trials. On each test trial, a single item was presented in the center of the display, and participants were asked to classify the item according to the channel the item was most likely to receive. No feedback was provided after their judgment. Following their response, participants were then asked to rate how confident they were about their response using a scale ranging from 1 (*"not at all" confident*) to 5 (*"extremely" confident*). Participants made classification and confidence responses at their own pace. At the end of each block, participants were told their cumulative accuracy during the block they just completed, as well as their accuracy during the preceding test block.

Results

Information sampling during selection learning. The aggregate distributions of selection participants' queries (i.e., their locations in stimulus space) in each of the eight training blocks are shown in Figure 6A. Note that sampling close to the true category boundary is often an adaptive selection strategy. After a little learning, items far from the boundary are unlikely to be misclassified and should be associated with more confident responses. In contrast, items near the boundary between the two categories are often associated with greater uncertainty. Thus, in the following analysis, we were interested in the distance of selections from the category boundary as a general measure of uncertainty-driven information selection.

In both the RB and II tasks, participants began by widely distributing samples over the stimulus space. Examination of individual participants' data revealed that many began by testing the boundaries of the space (i.e., the most extreme values on either feature dimension). Over time, participants were increasingly likely to sample more closely to the true category boundary, but the extent of this shift differed between the RB and II tasks. To quantify this change in sampling behavior, we measured the orthogonal distance of each sample to the true category boundary and computed the average of this distance for each training block (see Figure 6B). We performed a two-way analysis of variance (ANOVA) of average sample distance with task (RB/II) as a between-subjects factor and training block (1–8) as a within-subjects factor. There were significant main effects of task, $F(1, 440) = 12.19$, $p < .001$, and block, $F(7, 440) = 4.4$, $p < .001$. As seen in Figure 6B, sample distance was lower in the RB task than the II task, and decreased in both tasks over the course of training.

In addition to the main effects of task and block number, there was a significant interaction, $F(7, 440) = 2.69$, $p = .009$. Although average sample distance decreased over time in both tasks, participants were clearly better able to sample along the true category boundary in the RB task than in the II task. For example, in the RB task, average sample distance was significantly smaller than expected from a random-sampling strategy (dotted lines in Figure 6B) by the third training block (one-tailed t test), $t(28) = -2.21$, $p = .017$, and in all subsequent blocks. In the II task, sample distance never dropped below the level expected from a random-sampling strategy (one-tailed t tests, all ps $> .05$). However, in the II task, there were individual participants who consistently selected items along the category boundary. Figure 6A shows two individual participants from both the RB and II tasks.

Classification. Responses during test blocks were scored according to whether the participant identified the correct category of each test item (as determined by the true category boundary). Three participants (one each in the RB/S, RB/Y2, and II/S conditions) were excluded from further analysis because their overall performance (averaged across blocks) was more than three standard deviations below the mean of their group. Overall accuracy across tasks and conditions is shown in Figure 7A. We performed a two-way ANOVA with task type (RB/II) and training condition (S/R/Y1/Y2) as between-subjects factors. There was a significant main effect of task, $F(1, 229) = 2.28$, $p < .001$, with performance higher in the RB task overall, as well as a main effect of training

[4] In this design, reception participants are not matched to selection participants in terms of perceptual-motor demands (e.g., precisely adjusting the stimulus before observing the category label). However, pilot data suggested that attempting to equate this interaction (e.g., having reception learners adjust the stimulus to a prespecified "target") made learning much more difficult for the reception group, potentially adding to any advantage for selection-based learning.

A further difference between selection and reception was the timing of their response relative to the category feedback (selection learners clicked the mouse to show the label, whereas reception learners pressed a button after the label was shown to confirm its identity). Out of concern that this procedural difference might contribute to our results, a smaller follow-up control experiment was run in which the same response was required to reveal the category label in both the selection and yoked conditions. However, the overall pattern of performance was similar to those reported in the present study.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

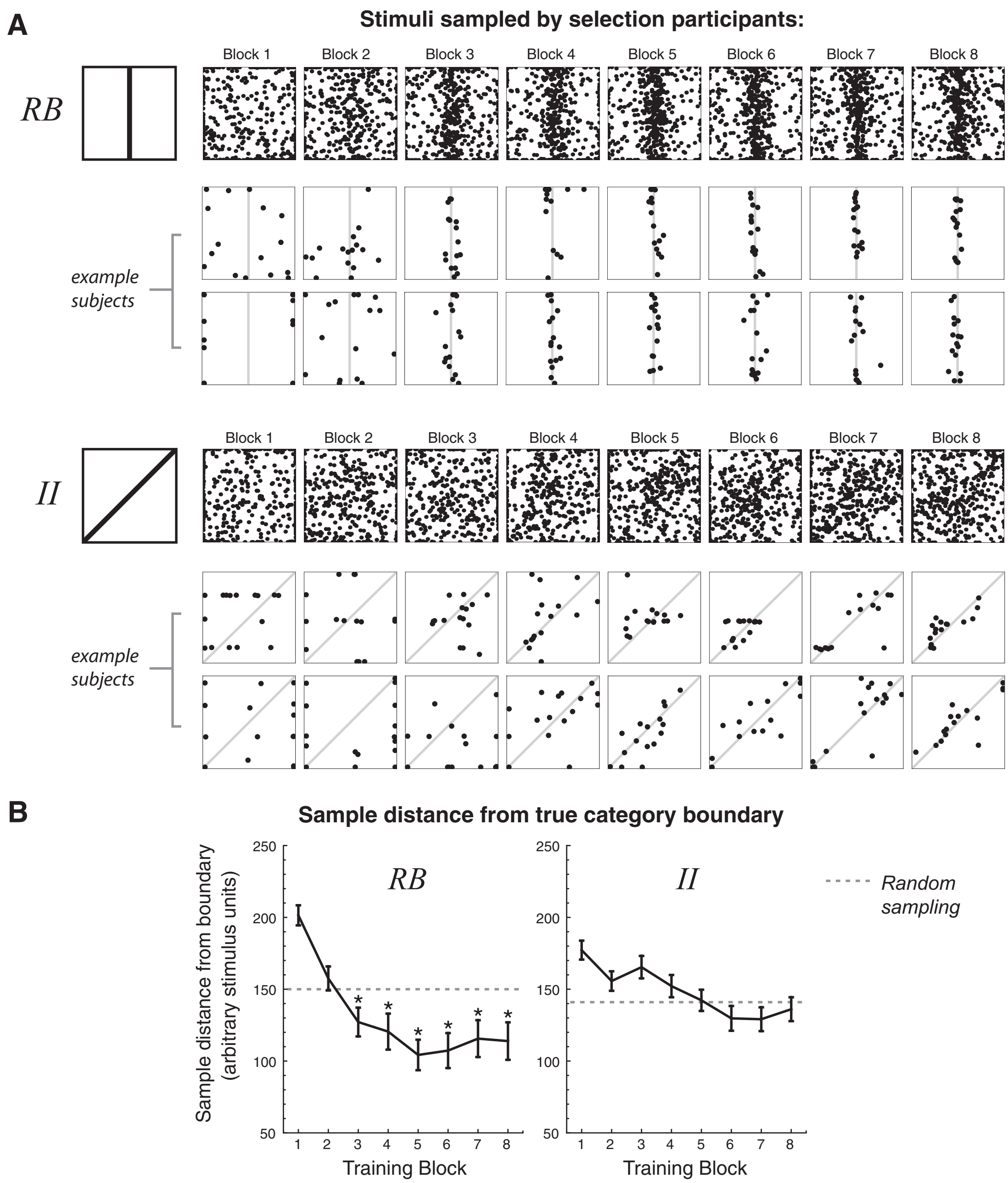

Figure 6. A: Distributions of samples chosen by selection participants in the rule-based (RB) and information-integration (II) tasks. Each point specifies the size and angle of an antenna that a participant selected (stimulus spaces have been rotated so that the decision boundaries align across participants). Shown below each composite are the selections made by two participants whose selections were closest to the true boundary, on average (relative to the rest of the participants in their group). B: Average distance of participants' samples from the optimal decision boundary by training block (black line). Dotted lines show the average distance expected from a random-sampling strategy, and stars denote blocks in which sample distance was significantly smaller. In the RB task, participants sampled significantly closer to the true category boundary than expected by a random strategy by the third block. In the II task, sample distance decreases over time, but never drops below the level expected from chance. The difference in chance responding between the two tasks results from the orientation of the optimal decision boundary in the space. Average sampling distances that appear "worse" than random in the early blocks is the result of a bias that some participants showed toward sampling at the extreme edges of the space. Error bars show the standard error of the mean.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

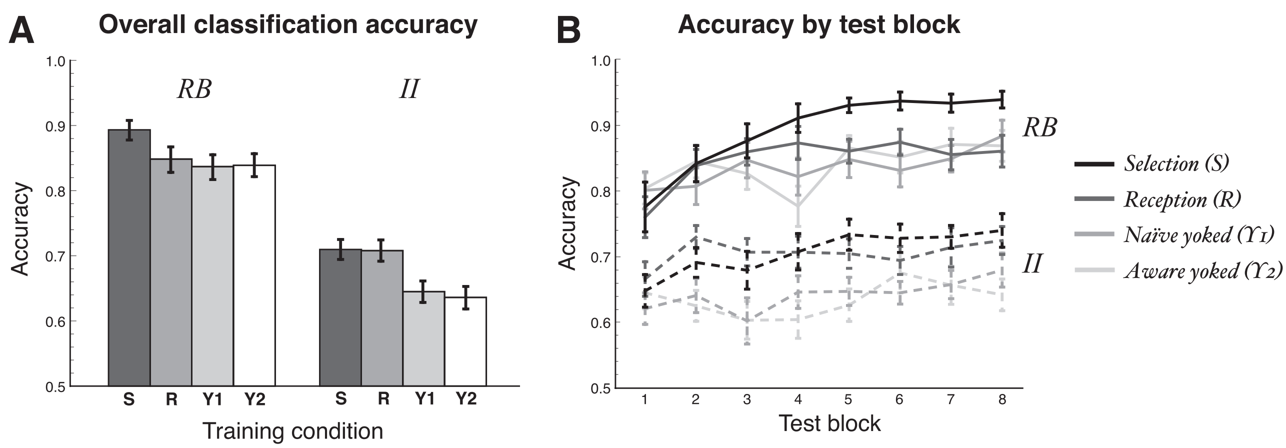

Figure 7. A: Overall classification accuracy for each condition averaged across all test blocks. B: Mean accuracy for each condition as a function of test block (learners of the rule-based [RB] structure are depicted with solid lines; the information-integration [II] structure is shown as dashed lines). Error bars show the standard error of the mean in both plots.

condition, $F(3, 229) = 6.5$, $p < .001$, with selection learners more accurate than both yoked conditions: Y1, $t(116) = 2.56$, $p = .01$; Y2, $t(115) = 2.71$, $p = .008$ (all other pairwise comparisons $p > .05$). There was no interaction between task and training condition, $F(3, 229) = 1.3$, $p = .27$.

In the RB task, overall accuracy was marginally higher in the S condition than in the R condition, $t(57) = 1.82$, $p = .07$, and significantly higher than both yoked conditions: Y1, $t(57) = 2.33$, $p = .02$; Y2, $t(56) = 2.34$, $p = .02$. There was no difference between accuracy in the R condition and either yoked condition: Y1, $t(58) < 1$; Y2, $t(57) < 1$, and no difference between the two yoked conditions, $t(57) < 1$. As shown in Figure 7B, all four training groups perform at the same level in the early part of the task, but as the task progresses, an advantage emerges for the selection learners over all three reception groups. By the third block, RB selection learners reached the same level of performance as achieved by the random-reception participants in Block 8 (see Figure 7B). Thus, it took the reception learners 2.7 times as long to achieve the same level of performance.

Within the II task, there was no difference between the selection and random-reception groups, $t(57) < 1$, but performance was greater in these two conditions when compared with either Y1: S, $t(57) = 3.09$, $p = .003$; R, $t(58) = 2.72$, $p = .008$, or Y2: S, $t(57) = 3.35$, $p = .001$; R, $t(58) = 2.97$, $p = .004$. As in the RB task, there was no difference between the two yoked training groups, $t(58) < 1$.

Because yoked performance was lower than that of selection learners in both tasks, we were interested in whether individual yoked participants benefited from being paired with high-performing selection participants. In both tasks, however, there was no correlation between selection learners' overall accuracy and the accuracy of yoked learners linked to the same training data, regardless of whether the yoked learners were naïve: RB, $r = -.03$, $p = .39$; II, $r = -.26$, $p = .15$, or aware: RB, $r = -.1$, $p = .34$; II, $r = .18$, $p = .24$. This runs counter to the prediction of the Bayesian model introduced in the introduction, which holds that the composition of the training data drives performance (and thus, accuracy should be correlated for pairs of participants who view the same items).

We next tested whether there were differences in confidence ratings or reaction times (RTs) during classification trials. Although there was a main effect of task type (II learners were both less confident and slower to respond), there were no significant effects of training condition. Due to the lack of a difference between training conditions, confidence ratings and classification RT were not analyzed further.

Training trial duration. Due to the need for selection participants to interact with stimuli during learning, we expected there may be differences in the duration of training trials between conditions, which could potentially play a role in the differences in performance described in the previous section. However, the pattern of results cannot account for the accuracy differences observed between conditions. A two-way ANOVA on median trial duration with task type (RB/II) and training condition (S/R/Y1/Y2) as between-subjects factors confirmed a significant main effect of training condition, $F(3, 229) = 177.6$, $p < .001$, and a main effect of task type with a shorter duration in the RB task overall (RB: $M = 3.1$ s, $SD = 2.1$; II: $M = 3.6$ s, $SD = 2.5$), $F(1, 229) = 5.2$, $p = .02$, but no interaction, $F(3, 229) < 1$. Duration was not significantly different in the S conditions between the two tasks (RB: $M = 5.5$ s, $SD = 1.4$; II: $M = 5.9$ s, $SD = 1.6$), $t(56) = 1.0$, $p = .3$, despite the poorer performance for II selection learners. Similarly, although duration was shorter in the R conditions overall, it did not predict the changes in performance between task and training condition (RB/R: $M = 1.3$ s, $SD = 0.4$; RB/Y: $M = 1.8$ s, $SD = 0.7$; RB/Y1: $M = 1.4$ s, $SD = 0.6$; II/R: $M = 1.6$ s, $SD = 0.7$; II/Y: $M = 2.3$ s, $SD = 1.9$; II/Y1: $M = 1.6$ s, $SD = 0.6$).

Relating sampling behavior and learning. Our next goal was to examine the relationship between sampling decisions and task performance. Specifically, we tested whether a participant's classification accuracy was related to how closely his or her training samples fell to the objective category boundary. For selection learners, we found that mean sample distance from the true category boundary was negatively correlated with overall test performance in both the RB ($r = -.60$, $p < .001$) and II ($r = -.54$, $p = .003$) tasks (see Figure 8A), showing that participants who sampled closer to the category

[5] Due to their equal test performance, naïve- and aware-yoked groups were combined for this analysis. However, a similar relationship is found between sample distance and accuracy when the groups are considered separately.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

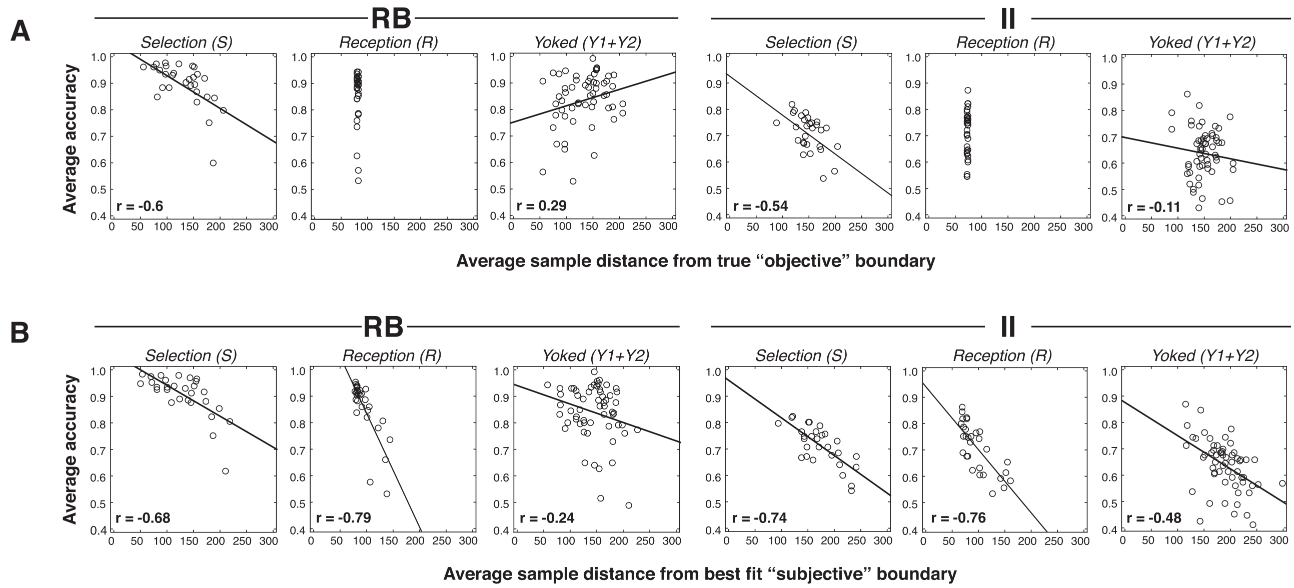

Figure 8. A: Relationship between the average distance of samples from the true category boundary and participants' overall accuracy. Samples closer to the true boundary were associated with higher accuracy in selection but not yoked learners. In the random-reception (R) groups, the samples shown to all participants had the same average distance from the boundary. B: When sample distance is measured relative to a learner's best fit boundary on the previous test block, it is correlated with overall accuracy for all groups in both tasks. RB = rule based; II = information integration.

boundary were more accurate overall. Interestingly, the same relationship was not found for yoked learners who were trained with the same stimuli.[5] In the RB task, there was a positive correlation between sample distance and performance ($r = .29$, $p = .04$), whereas in the II task, there was no correlation ($r = -.11$, $p = .29$). As is visible in Figure 8A, yoked learners who received data that fell closer to the category boundary were often among the worst performers in their group (this is particularly clear in the RB task), whereas the same training data were associated with higher performance in the selection participants. This pattern is particularly challenging to explain because it suggests that training sequences that helped one learner perform well actually impaired the performance of a separate yoked learner. It is consistent, however, with the predictions of a hypothesis-depending sampling bias, as illustrated in Figures 3 and 4.

Decision-bound analysis of test-block responses. In a final analysis, we attempted to characterize changes in participants' beliefs about the category boundary during learning. Our theory of a hypothesis-dependent sampling bias has assumed that learners preferentially select items that fall near the category boundary that they are currently considering. To the degree that a participant's classification responses during test blocks reflect their current understanding of the categories, we can use these responses to estimate their current hypothesis. To this end, we fit linear decision boundaries to each participant's responses in each test block. Measuring changes in the parameters of the best fit bounds from block to block provides a coarse description of the frequency and magnitude of shifts in beliefs over time. In addition, measuring the distance of each selected training item from a participant's own best fit boundary (as opposed to the objectively true boundary) may be more informative about how information-sampling behavior relates to dynamic changes in the participant's beliefs.

In each test block, a participant provided category labels for a set of items that were uniformly distributed throughout the stimulus space. We found the best fitting linear decision boundary for each block of test responses from each participant. A decision bound can be described by three parameters: θ, the angle of the decision bound in space; b, the bias, or offset from the center of the space; and σ, the determinism of the boundary. The likelihood that a given observation x belongs to Category A is a sigmoidal function defined by the parameters $\{\theta, b, \sigma\}$:

$$P(x^t = A|\theta, b, \sigma) = \frac{1}{1 + exp\left(-\sigma(x_1^t \cdot cos(\theta) + x_2^t \cdot sin(\theta) - b)\right)}, \tag{2}$$

where x_i^t gives the observed value on dimension i on trial t. Because the classification is binary, $P(x^t = B|\theta, b, \sigma) = 1 - P(x^t = A|\theta, b, \sigma)$. The likelihood of a particular set of labeled observations $D = \{x^1, \ldots, x^n\}$ is given by $P(D|\theta, b, \sigma) = \prod_t P(x^t|\theta, b, \sigma)$. Given this likelihood function, the best fitting parameters were found using a standard optimization procedure that maximizes the log likelihood of each set of responses. Qualitatively speaking, most participants' test-block responses were well characterized by a linear decision boundary of this form. We compared the Akaike's information criterion of the decision-bound model with that of a random-response rule in order to find the number of test blocks that were best fit by the decision-bound model. Figure 9A shows the proportion of participants for whom a given number of blocks were best fit by the decision-bound model compared with random responding. In general, the decision-bound model provided a better fit, particularly for all RB conditions, in which approximately 80% of participants had all blocks better accounted for by the decision-bound model. In the II condition, an increased number of blocks were best fit by the random model, but a

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

majority of participants still had seven to eight test blocks best fit by the linear decision model.

Figure 9B shows the distribution of best fit values of θ for those blocks in which the decision-bound model provided a better fit than random responding, with the black bars indicating the interval that contains the true category boundaries for each task.[6] In the RB task, there was a clear preference for unidimensional rules, with strongly diagonal rules providing the best fit in a small proportion of blocks (S: 1%, R: 4%, Y: 4%). In the II task, there was a higher proportion of diagonal rules (S: 21%, R: 23%, Y: 18%), but unidimensional rules still accounted for a greater number of test blocks overall (S: 33%, R: 31%, Y: 37%).

Sample distance from best fit subjective boundary. In our earlier analysis (see Figure 8A), we found that sampling closer to the true category boundary was associated with higher performance in selection, but not yoked, learners. One explanation for this divergence is that samples are more useful with respect to the selection learner's current beliefs (i.e., they are produced by a hypothesis-dependent sampling bias). Accordingly, the distance of samples from a learner's current "subjective" decision boundary may better predict how they learn from those observations. To evaluate this idea empirically, for each participant we measured the average distance of selections on each training block (excluding the first) to the best fit decision boundary for the previous test block. Using this subjective measure, we again found that sample distance was strongly correlated with overall performance for selection learners in both tasks (see Figure 8B; RB: $r = -.69$, $p < .0001$, II: $r = -.74$, $p < .0001$). Unlike the previous analysis, however, there was a trend toward the same relationship for yoked participants in the RB task ($r = -.24$, $p = .06$) and a significant correlation in the II task ($r = -.48$, $p = .0001$). In addition, although objective sample distance was fixed across reception participants, using subjective distance we found the same relationship in both RB ($r = -.79$, $p < .001$) and II ($r = -.75$, $p < .001$) tasks. We next tested whether subjective distance differed between participants who observed the same training data depending on the task and their condition (selection vs. yoked reception). A two-way ANOVA on subjective distance with task and condition as between-subjects factors revealed significant main effects of both task, $F(1, 173) = 58.2$, $p < .001$, and condition, $F(1, 173) = 5.93$, $p = .015$, but no interaction, $F(1, 173) < 1$. Subjective distance was smaller for selection learners than for yoked learners, as might be expected because selection learners were making the sampling decisions themselves. In addition, overall subjective distance was smaller in the RB task than in the II task, further suggesting that sampling was less effective in the II task.

Variability in decision rules during learning. Finally, we measured changes in the best fit parameters from block to block in order to better understand how participants' beliefs shifted over time. For example, given that θ describes the importance attributed to either feature dimension, changes in θ signal exploration of different forms of rules. Changes in *b*, however, suggest refinement of the location of the boundary in space (although changes in both parameters can occur simultaneously). Variability in the value of σ, unlike the other two parameters, does not reflect adjustments to the location of the boundary in space, but rather changes in the determinism of responses.

The magnitude of changes in best fit parameters was measured between pairs of consecutive test blocks and averaged over early ($1 \rightarrow 2$, $2 \rightarrow 3$, $3 \rightarrow 4$) and late ($5 \rightarrow 6$, $6 \rightarrow 7$, $7 \rightarrow 8$) transitions. Because the distribution of these values violated the assumptions of standard *t* tests, we used the nonparametric Wilcoxon rank sum test to evaluate differences in variability between conditions. Figure 10 provides a compact summary of these analyses. First, consider the variability of θ (see Figure 10, left) and *b* (see Figure 10, middle). Overall, variability in both parameters was greater in the II task than in the RB task (θ: $W = 11861$, $p < .001$, *b*: $W = 10478$, $p < .001$), suggesting that II learners were more likely to make large changes to their classification decision strategy from one block to the next.

In the first half of the RB task, there was no systematic difference in the variability of either θ and *b* between conditions. In the latter half of the task, however, variability in the same parameters is significantly lower in the S condition than both reception training conditions for both θ (R: $W = 296$, $p = .04$, Y: $W = 575$, $p = .01$) and *b* (R: $W = 222$, $p = .001$, Y: $W = 303$, $p = .04$), whereas variability is not different between R and Y conditions. This suggests that by the second half of the RB task, selection participants had learned the correct form of the rule and made smaller adjustments from block to block than participants in the reception conditions. Variability of θ and *b* in the II task followed a similar pattern for the S and R conditions. The major difference in this task was that random-reception learners show less variability in *b* in early blocks than the other conditions (S: $W = 605$, $p = .02$, Y: $W = 1170$, $p = .02$), and equal variability to the S condition in late blocks for both θ and *b* (consistent with their classification performance being the same as the S condition).

Variability in σ (see Figure 10, right) indicates differences in how deterministic responses were from block to block (e.g., transitioning from random classification on one block to the use of a sharp, deterministic rule on the subsequent block). Overall variability in σ (transformed to log scale) was significantly higher in the RB task than in II task ($W = 31{,}002$, $p = .05$). However, there were no differences in the variability of σ between training conditions in either early or late blocks.

The key point from this analysis is that poorer performance for reception learners in the RB task and the yoked group in the II task seemed to be related to an ongoing search for the correct form of decision boundary. In particular, the pattern suggests that yoked behavior was marked by larger, more frequent shifts in decision bounds throughout the experiment (as may be expected from ongoing hypothesis testing). Notably, the average change in θ for yoked learners in the latter half of the II task is approximately equal to a shift from a unidimensional rule to a diagonal rule (marked by the horizontal line in the plot for θ in the first panel of Figure 10). In contrast, changes in both θ and *b* are smaller in late blocks for selection learners, consistent with their higher overall accuracy.

[6] The θ parameter determines the orientation of the decision boundary in the stimulus space. A value of $\theta = 0$ implies a vertically oriented decision boundary (as illustrated along the *x*-axis in Figure 9B). This parameter can be viewed as the relative weight given to either dimension in the task in which the weight for dimension one is $w_1 = cos(\theta)$, and the weight for dimension two is $w_2 = sin(\theta)$.

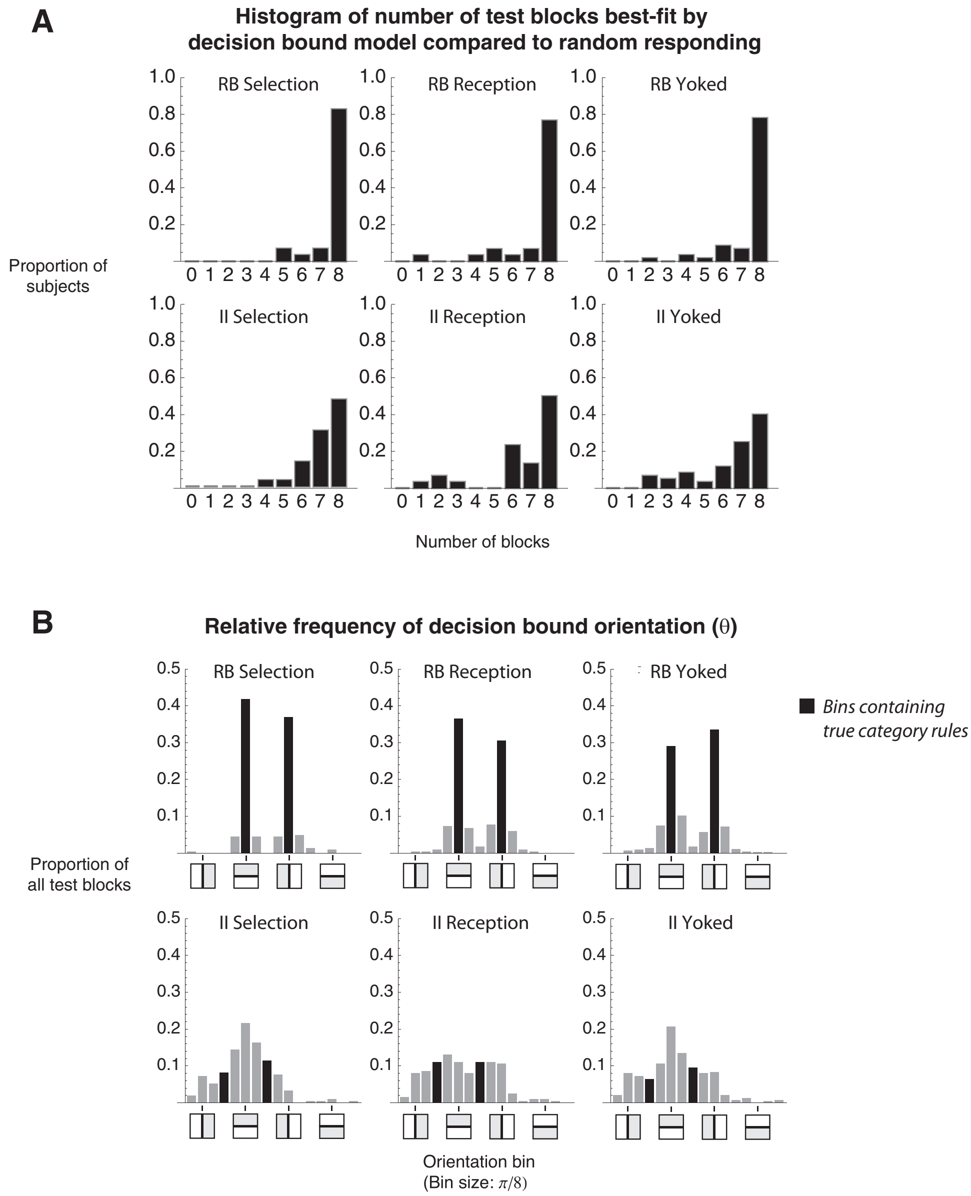

Figure 9. A: Histogram showing the number of test blocks best fit by the decision-bound model compared with a random-response rule. In all rule-based (RB) conditions, more than 80% of participants did not have a single block that was better accounted for by random responding. In the information-integration (II) task, there were a greater number of blocks that were best fit by random responding, but in all conditions more than 60% of participants had one or fewer blocks of this kind. B: Relative frequency histogram of the best fit value of θ for all test blocks, divided by condition. The black bars indicate the bins containing the orientation of the true category boundaries in each task, whereas the gray bars correspond to all other orientation bins. In the RB task (top row), most decision bounds correspond to a unidimensional rule. In the II task (bottom row), unidimensional rules still account for a large proportion of test blocks across conditions.

Discussion

There are a number of key observations from the experiment. First, we found a main effect of category structure, with participants in the II task performing more poorly overall. This result is consistent with previous category learning studies that explored the RB and II distinction under observational learning conditions (Ashby et al., 2002). This overall pattern likely reflects biases in how people approach such tasks (preferring simple “verbalizable” rules over more complex decision boundaries). Because the number of training trials in our experiment was relatively low, it is unlikely that incremental procedural learning processes contributed to performance (in fact, this is a prediction of the dual-system COVIS framework; Ashby et al., 1999).

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

Second, we found that selection learners were more accurate than random-reception participants, but only in the RB task. Although this result was surprising (one might have expected that selection learning would provide a performance boost for any task), it is consistent with previous work that has suggested that selection learning is less effective for more complex problems (Enkvist et al., 2006; Schwartz, 1966). Importantly, any theory that argues that selection simply leads to greater "engagement" with the task would have trouble explaining why there would be a differential advantage based on problem type.

Third, we found a strong relationship between selection decisions and performance in both RB and II tasks. Our theory of the hypothesis-dependent sampling bias suggests that selection learners should differentially query regions in the stimulus space where they were most likely to commit classification errors (i.e., near the boundary). Consistent with this idea, selection learners in the RB task showed a clear preference for items that were close to the true category boundary (see Figure 6) and were close to the boundary that they were considering on the previous test block.

This sampling strategy helps to explain why selection participants in the RB task outperformed participants in the R condition who received samples from predefined distributions. Even though the overall average distance of their samples from the boundary was relatively low, random-reception participants were less likely to observe training items close to the critical boundary than many selection participants (e.g., consider the sampling strategies of the two individual participants shown in Figure 6). Given the large literature on confirmation bias, it is notable that RB selection participants were able to sample effectively within the context of a novel, abstract task, supporting previous findings of efficient sampling in other problems (Castro et al., 2009; Gureckis & Markant, 2009; Oaksford & Chater, 1994). In contrast, selection participants in the II task were less likely to sample near the true category boundary and had lower accuracy than the same condition in the RB task. As in the RB task, however, the tendency to do so was related to the learner's performance, and there were a few highly effective II samplers who also achieved higher accuracy relative to the rest of their group (see the bottom two examples in Figure 6).

However, any advantage for selection learners cannot be explained by a difference in training data alone. Most striking is the finding that yoked participants in both tasks were less accurate than the selection group despite learning from the exact same observations. Indeed, the yoked participants who observed what might be considered the most objectively useful training data (as measured by distance from the true category boundary) were among the worst performers, particularly in the RB task. If selection and yoked learners are assumed to update their beliefs through a common process (as would be predicted by most existing models of human categorization, including the Bayesian model described earlier), then this strong pattern of divergence is unexpected.

The key to understanding this pattern was our analysis of how new observations were related to estimates of participants' decision boundaries. In all tasks, performance was negatively related to the distance of the observed samples to the individual's decision boundary on the previous block. This suggests that learners are better able to learn from samples that fall in regions they currently consider uncertain. Because yoked and selection participants will be considering different hypotheses at any point in time (particularly early in the task), this can lead to performance divergences. This overall data pattern is a core prediction of our theory of the hypothesis-dependent sampling bias.

The results of the decision-bound analysis also provide insight into the basis for participants' errors during learning. For example, it is not the case that yoked learners failed to classify items systematically or to change their beliefs following poor performance in a test block. Instead, their response variability suggests that, on the whole, yoked participants continued to search for the correct decision bound but were less likely to acquire (and maintain) it than selection learners. This is consistent with the idea that participants were engaged in sequential hypothesis testing (Gregg & Simon, 1967; Nosofsky & Palmeri, 1998; Nosofsky, Palmeri, & McKinley, 1994; Trabasso & Bower, 1964). Even in the second half of the task, we found that some participants changed which dimension they used to classify exemplars, which is unexpected if participants were integrating over all previous training examples.

One possible explanation for this response variability is that participants only use a small number of recent observations to evaluate their belief about the category rule (consistent with the classic studies of hypothesis testing by Trabasso & Bower, 1964). This is also intuitively likely given that the category exemplars are highly similar to one another and easily confusable. A limited memory for past observations may heighten the importance of effective sampling, such that the most recent examples are used to judge the validity of the learner's existing belief about the category boundary.

Overall, the experimental results provide strong evidence for interactions between the mode of information sampling and the structure of the concept being learned. Moreover, they suggest that the difference between selection and yoked reception was not based simply on the act of selecting data, but rather the *kind* of data that is collected and whether it is useful to the learner. The advantage for selection learners depended on their ability to collect information close to their estimate of the category boundary, whereas the same information was less beneficial when observed by other participants. This finding is inconsistent with the idea that selection improves performance through increased "engagement" or another generalized change in the learning process. In general, it supports the idea that selection learning introduces a hypothesis-dependent sampling bias such that new observations are linked to the sequence of hypotheses entertained by the person who made them.

In the next section, we present a model based on the principles described earlier in the present article, allowing us to systematically investigate how information collection interacts with learning in our task. Our goal was to introduce a computationally explicit model that can account for our experimental results while also demonstrating how hypothesis-dependent sampling biases affect learning outcomes more generally.

Modeling Interactions Between Sequential Hypothesis Generation and Information Selection

At the broadest level, our theory asserts that people engage in sequential hypothesis testing (consistent with the ongoing shifts in response rules seen in the experiment) and that the selection of new observations is biased by the learner's current hypothesis (the hypothesis-dependent sampling bias). Given these assumptions,

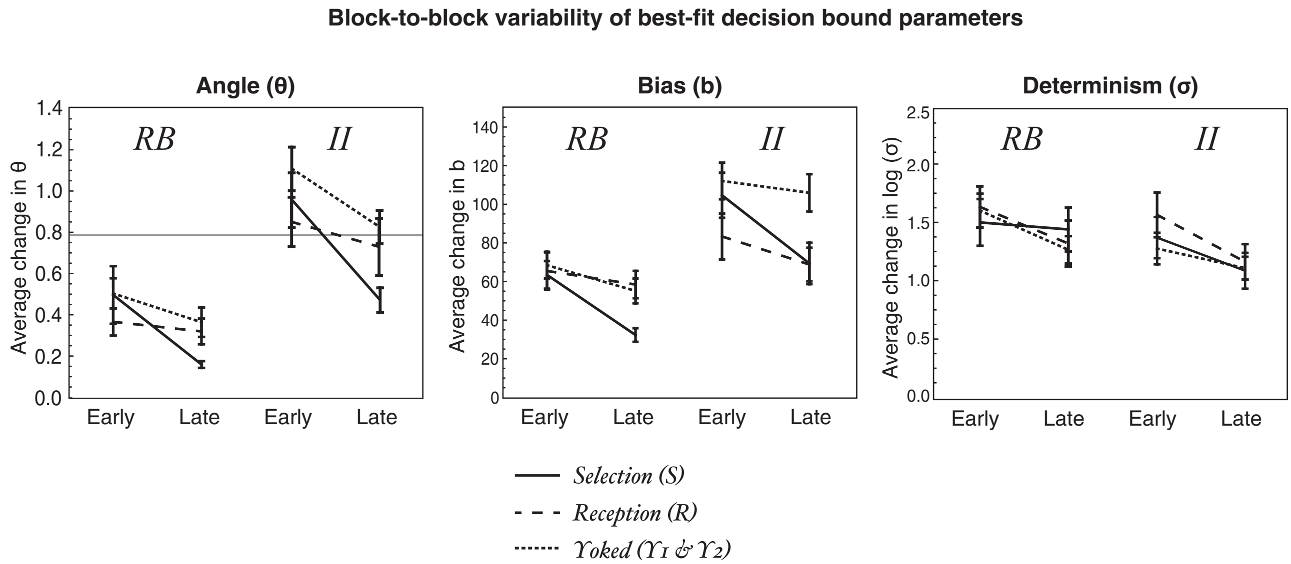

Figure 10. Variability of best fit decision-bound parameters between test blocks (measured as average difference in absolute value of each parameter between subsequent blocks). Variability in both θ and *b* was greater in the information-integration (II) task than in the rule-based (RB) task because participants were less successful at learning the diagonal rule. Variability between early blocks was the same for selection and yoked participants. In late blocks, variability decreased more for selection than yoked learners. Notably, in the II task, the average change in θ for the yoked group in the second half of the task was approximately equal to a change from a unidimensional rule to a diagonal rule (marked by the horizontal line). Variability in the best fit value of σ (transformed to log scale) was higher in the RB task than II task, but there were no differences between training conditions.

we show that selection and reception can lead to different learning outcomes without assuming any other differences between conditions.

Our simulations address two theoretical questions posed by the behavioral results. First, given the focus placed on reception learning in the literature, existing category learning models provide no explicit account of how different training conditions would produce our experimental results. In Simulation 1, we show that our model can recreate the pattern of results when the learning process is the same between conditions, and all that differs is the task and mode of information sampling.

Second, existing models have difficulty explaining the divergence between selection and yoked reception learners (because the sequence of training examples is identical in both conditions). Simply assuming selection learning is advantageous because of increased “engagement” or another generalized cognitive factor fails to account for the interaction between sampling behavior and performance that was observed in the experiment (i.e., under an engagement hypothesis, Figure 8A would show a main effect rather than an interaction). Our second simulation tests our hypothesis that this relationship, and the divergence between selection and yoked reception groups in general, is a direct consequence of a hypothesis-dependent selection process, and explores the conditions under which this kind of divergence is expected to emerge.

Description of the Model

There are five key principles governing the model. First, we assume that linear decision bounds are used to make decisions about category membership (consistent with evidence that learners have a preference for such rules early in a learning task). Second, the learner generates, and potentially switches to, a new hypothesis on each trial based on a stochastic process (consistent with models of sequential hypothesis testing). Third, we assume that the generation of new hypotheses is strongly biased by a prior preference for unidimensional rules. Fourth, we assume that a limited number of prior training examples may be stored in memory and used to evaluate the plausibility of a given decision bound. Finally, we assume that selection learners display a hypothesis-dependent sampling bias such that the instances they choose to learn about are tied to their current hypothesis in a systematic way. In our later simulations, we evaluate the contribution of each of these assumptions to the model’s performance.

Category rule representation. The model classifies items using simple decision rules (similar to Ashby, Paul, & Maddox, 2011; Nosofsky & Palmeri, 1998), and the goal of learning is to infer the rule that correctly classifies the most items. Each hypothesis is represented as a linear decision bound that assigns a probability of category membership to each item in the space using Equation 2. The model’s hypothesis on trial t is defined by two parameters, $h^t = (\theta^t, b^t)$, which control the position and orientation of the decision boundary in the stimulus space (for simplicity, we assume all decision bounds are deterministic).

Prior. The prior probability of a hypothesis h is defined as the joint probability of its two parameters, $p(h) = p(\theta)\, p(b)$. Previous research suggests that learners are strongly biased toward unidimensional rules along either dimension in the early stages of

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

learning (cf. Heller, Sanborn, & Chater, 2009). Accordingly, we defined a prior over θ, the angle of the vector corresponding to the decision boundary, which favored axis-aligned rules (see Figure 11). For a given value of θ, the relative distance r of the decision bound from the nearest orthogonal axes is given by:

$$r = \frac{2}{\pi}\left[\theta \; mod \, \frac{\pi}{2}\right], \tag{3}$$

which is bound between 0 and 1, with *mod* referring to the arithmetic modulo. r is assumed to have the following distribution:

$$r \sim Beta(\alpha, \alpha). \tag{4}$$

New values of θ are generated by randomly choosing a quadrant (defined by k, an integer between 0 and 3) and sampling a value of r, which then determine the value of θ:

$$\theta = \frac{\pi}{2}(k + r). \tag{5}$$

Under this prior, α acts as an abstract decision weight for the two stimulus dimensions. When $\alpha < 1$, there is a general preference for axis-orthogonal rules that involve only a single dimension (whereas $\alpha = 1$ implies no preference for rules of a particular orientation). A prior for unidimensional rules (i.e., when $\alpha < 1$) is consistent with a large body of empirical and theoretical work suggesting that learners find axis-aligned boundaries easier to learn and are most likely to be used early in learning (Ashby et al., 1998, 2002, 1999).[7] The prior over the bias term was a uniform distribution over the stimulus space.

Generating new hypotheses. With this representation of decision rules and prior, a Bayesian learner would evaluate the posterior probability of all possible hypotheses given a set of observations D according to Equations 1 and 2. However, given capacity limitations, it is unlikely that participants can simultaneously update and represent this entire hypothesis space. Instead, we assume that at any point in time, a learner uses a single hypothesis h^t as the basis for classification and that on each trial, they consider making a change to the current hypothesis. This makes our model more consistent with the large literature on sequential hypothesis-testing models of learning (Gregg & Simon, 1967; Trabasso & Bower, 1964; Trabasso et al., 1968) along with more recent variants such as RULEX (Nosofsky et al., 1994) or the explicit learning system in COVIS (Ashby et al., 1998).

The stochastic procedure for generating hypotheses is as follows: On each trial, one of the two parameters (θ^t, b^t) is randomly modified to create a proposal, h'. A free parameter p_θ sets the probability that θ is modified, in which case a new value θ' is simply generated from the prior. When b is modified, a new value is sampled from a Gaussian distribution centered on the current value $b' \sim N(b^t, s_b)$. By this mechanism, the learner considers either local changes in the position of the decision bound in the feature space (by adjusting b) or a change in the relative weight of the two features (by sampling a new value for θ).

After a proposal has been generated, the learner either adopts or rejects it on the basis of the acceptance ratio a, the relative posterior likelihood of the existing and proposal hypotheses (given by $a = \frac{p(D|h')p(h')}{p(D|h^t)p(h^t)}$, where $p(D|h)$ is the likelihood from Equation 2, and $p(h)$ the prior described in the previous section). If the posterior likelihood of the proposal is equal to or greater than that of the previous hypothesis (i.e., if $a \geq 1$), it is accepted as the new active hypothesis (i.e., $h^{t+1} = h'$). If the proposal results in a worse account of past data, it is accepted in proportion to the acceptance ratio. Otherwise, the current parameter estimate remains unchanged.[8]

The psychological demands of this process are low: On each trial, the learner must simply generate a new hypothesis and judge its quality relative to the single existing hypothesis. Sometimes a proposal is accepted and becomes the active decision bound, and other times it is rejected as being a worse account of recent training data. This procedure is closely related to simple Win-Stay-Lose-Shift heuristics for sequential hypothesis testing (Trabasso et al., 1968). Note that in comparison to the simpler model described in the introduction, this procedure does not require that a consistent hypothesis is found on each trial (an assumption that would generally lead to higher performance than observed in the experiment, particularly in the II task).

Limited memory. Our analysis of how participants changed their decision rules over the course of the task suggested that learners typically do not optimally integrate past observations, which is contradicted in particular by large changes in the form of the decision rule late in training. In the model we use the parameter n to specify the number of recent observations that are used to evaluate the likelihood of a hypothesis and (based on our empirical results) expected that n was likely to be relatively low for our participants.

A small memory capacity for previous observations leads to greater overall variability in the decision bound over the course of learning. For example, given a prior favoring unidimensional rules and few observations, the estimate of θ will tend to bounce between different modes of the hypothesis space. When n is low, convergence on the correct hypothesis will depend on the informativeness of the most recent training samples. When n is high, the model will tend to learn more quickly and have less variability in its decision rules.

Hypothesis-dependent sampling bias. Finally, we assumed that selection learners exhibit a hypothesis-dependent sampling bias whereby they choose to query instances that fall along their current decision bound. This assumption is supported by our empirical results in Figure 6 and 8B, which suggest preferential sampling of examples near the learner's current estimate of the boundary between the two categories. In addition, this strategy can

[7] Specifically, our approach is most similar to COVIS (Ashby et al., 1998), in which an explicit rule-generating system competes with an implicit exemplar-based system to produce responses. As mentioned earlier, in prior work with these types of tasks, learners appear to rely initially on the explicit system, with a bias toward using unidimensional rules for classification. This would be consistent with a computational level inductive bias toward simpler rule structures (Feldman, 2000; Kemp, 2012). Our prior captures this preference by assigning higher a priori likelihood to rules that involve attention to a single dimension.

[8] This procedure is identical to an algorithm in the statistics literature known as the Metropolis-Hastings (MH) algorithm, a form of Markov-Chain Monte Carlo (Metropolis & Ulam, 1949). This relationship draws our approach in line with recent efforts in developing "rational process models," which approximate Bayesian inference using hypothesis space sampling approaches (Brown & Steyvers, 2009; Vul, Goodman, Griffiths, & Tenenbaum, 2009a; Vul, Tenenbaum, et al., 2009b).

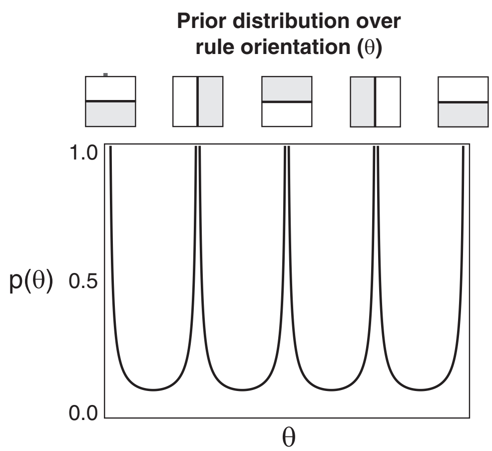

Figure 11. The distribution defining the prior bias for axis-aligned (unidimensional) rules in the model. Each peak corresponds to a different rule represented by the stimulus space divided into a white region for category A and a gray region for category B.

be motivated normatively by many theories of rational information acquisition (Nelson, 2005; Oaksford & Chater, 1994) and by research in machine learning, which emphasizes margin sampling (a learning strategy of sampling at the boundary of a classifier). According to our theory, the magnitude of the sampling bias—how close to the boundary samples are selected—will affect the relative performance of the selection and yoked reception learners.

On each trial, a new selection $\mathbf{x}$ is sampled from a region proximal to the current decision bound according to the following:

$$\begin{aligned} d_1 &\sim N(b^t, S) \\ d_2 &\sim U(0,1) \\ \mathbf{x} &= d_1\mathbf{u}_1 + d_2\mathbf{u}_2 \end{aligned} \tag{6}$$

where $\mathbf{u}_1$ and $\mathbf{u}_2$ are unit vectors that are orthogonal and parallel to the current decision bound, respectively. As a result, S controls the average orthogonal distance of new selections from the current hypothesis, whereas the location of those selections in the direction parallel to the current bound is uniformly distributed. Changes in the S parameter correspond to the difference between "strong" and "weak" hypothesis-dependent sampling biases that may vary between individuals or groups.

Simulation 1: Fitting Participants' Classification Performance

Our first goal was to evaluate whether the model could capture the overall pattern of results found in our experiment. Five hundred runs of selection, reception, and yoked reception models were trained in both RB and II tasks. Selection models generated new observations on each trial (using Equation 6), but were otherwise identical to reception models, with the exact same parameter settings shared across all three groups and for both the RB and II tasks. For each run of the selection model, a paired yoked model was given the same sequence of training examples. Reception models were trained with data generated from the same distributions as in the behavioral experiment. For all training conditions, the model was tested on a uniform grid of items on the same trials as in the experiment.

Average classification accuracy in each condition was compared with the human data from our experiment using root-mean-square error (RMSE). Using standard optimization procedures (grid searches along with Nelder-Mead simplex algorithm), we sought a single set of parameters that minimized the RMSE score. Table 2 shows the values of the best fit parameters. The results (see Figure 12B) capture the overall pattern of results from our experiment. First, accuracy was greater in the RB task than in the II task in all conditions. This effect is dependent on two main assumptions in the model: a prior bias toward unidimensional rules and a small memory for recent observations. In the RB task, a greater propensity for generating unidimensional rules (see the Prior section) allows the model to converge on the correct form relatively quickly, and over the course of learning refine its exact location by adjusting the bias term. Once the model has discovered the correct RB rule, relatively few instances are then necessary to maintain that rule.

In the II task, the model is less likely to generate the correct diagonal rule as a result of the unidimensional bias. Moreover, even when that rule is adopted at some point during training, the small number of observations held in memory is less effective at maintaining it when it is compared with a unidimensional alternative (which is more likely according to the prior). As a result, throughout learning, the active hypothesis alternates between modes of the posterior corresponding to suboptimal axis-orthogonal rules. Consistent with our behavioral results, the model *can* successfully acquire the correct II rule, but it is less likely to maintain that hypothesis over time.

The difference between these two tasks is also reflected in the pattern of selections generated (see Figure 13A). Like our behavioral results, the selections begin widely distributed in both tasks. As the model converges on the correct type of rule in the RB task, the selections become tightly clustered around the true category boundary, whereas in the II task, the pattern remains dispersed throughout learning (leading to a lower average distance from the true boundary for the RB task, as shown in Figure 13B). Note that, because the degree of sampling bias was fixed for all models in this simulation, this aggregate pattern arises from differences in the sequence of hypotheses generated in each task rather than a difference in the ability to select useful information.

The selection model performed better than the random-reception model in the RB task. As predicted, this occurs because the selection model generates samples close to the true category boundary, whereas the reception model is limited by the distribution of training samples. Inspection of the sequence of hypotheses showed that the reception model had higher variability in the bias term during learning, showing that the training data were less effective at maintaining a hypothesis in the center of the stimulus space. Notably, the difference between conditions does not occur in the II task, in which the model is biased against generating the correct form of rule in either training condition.

Finally, the simulation captures the divergence between selection and yoked reception learners in both tasks, despite sharing the same training data and identical parameter settings. This divergence arises directly from the model's stochastic process of generating new hypotheses during learning, such that a single obser-

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

vation may differ in how it influences the acceptance of new decision bounds (in the next simulation, we provide a systematic demonstration of when this divergence arises).

The model captures the pattern of results from the experiment and is particularly well matched to the RB task. The largest discrepancy is for the II yoked reception group, where the model predicts higher accuracy than was observed empirically. However, the model's performance in this condition is still consistently below that of the II selection group throughout learning (see Figure 12, right panel). Indeed, performance of the model during the last block of training matches the empirical results quite closely.

The main conclusion of this simulation is that a simple interaction between a single-hypothesis search process and different modes of information sampling can affect learning outcomes in the same manner observed in our experiment. Although this simulation provides a "proof of concept" of how a hypothesis-dependent sampling bias can influence learning, the key principles in the model are well motivated by a large body of work on hypothesis testing and category learning. The fit of the model would undoubtedly improve by allowing parameters to differ between individuals or groups in the experiment. However, our emphasis here is not on exactly matching the human data but on showing how the qualitative effects emerge even under idealized situations. In addition, we do not entirely reject the idea that there may be general attentional or motivational differences between conditions, but instead use our simulation results to illustrate how sampling and learning can interact to strongly influence the patterns of learning in addition to these factors. However, an important question is how the parameters each contribute to fitting the pattern of results. This is an issue we explore in more detail in Simulation 2.

Simulation 2: Diverging Outcomes due to Hypothesis-Dependent Sampling Bias

Whereas Simulation 1 shows that the model can capture the overall effects of training condition and category rule, a key aspect of our theory is that a hypothesis-dependent sampling bias causes a divergence in outcomes between selection and yoked reception learners. Note that in our experiment, performance among selection learners was correlated with the distance of their samples from the true category boundary (see Figure 8). In our model, the extent of this sampling bias is controlled by the S parameter (the standard deviation of a Gaussian centered on the current hypothesis). For our second simulation, we systematically varied the S and n parameters to evaluate the effects of the sampling bias under different conditions.

The results (presented in Figure 14, first three columns) show the effect of changing S at three different memory capacities (4, 8, and 128). When selections are generated randomly (leftmost column), performance is equivalent for both selection and yoked groups. As S decreases, new selections become increasingly biased by the selection learner's hypothesis. Consistent with our finding of a strong relationship between sample distance and accuracy, this decrease in distance from the current decision bound improves performance for the selection model, while the same change in the distribution of samples leads to lower yoked performance. Assuming individual differences in the S parameter across participants would thus explain the correlations between performance and sample distance from the boundary reported in Figure 8 for both selection and yoked learners (i.e., being yoked to a participant with a high sampling bias would lead to lower performance for you, but higher performance for them).

The divergence in accuracy is largest at low memory capacities (4 and 8) but still present even when the model stores all of the training data (n = 128). Note that although a low value of n was optimal for fitting our experimental results in Simulation 1, this shows that the predicted divergence is not dependent on a small memory capacity. Even when using all of the training data, the model benefits from testing its current hypothesis because its selections facilitate more efficient search of the hypothesis space. For example, when the current hypothesis is incorrect, strongly biased samples encourage the adoption of a new hypothesis. In combination with the one-dimensional model described in the introduction, these results further show that selection can be advantageous by virtue of its interaction with the hypothesis-generation process across a variety of conditions.

What happens when there is a hypothesis-dependent sampling bias, but new hypotheses are generated completely randomly? This result is shown in the rightmost column of Figure 14. Here a strong bias does not lead to any benefit for selection groups relative to their yoked counterparts, for the simple reason that those selections do not affect which hypotheses are generated next.

Although this simulation is a simplification of many aspects of the behavior seen in our experiment (e.g., the extent of sampling bias may have varied over time), it provides a mechanistic understanding of how selection, in combination with a stochastic, local hypothesis-generation process, can affect the speed of learning. In addition, the key principles of the model capture not only overall trends in accuracy between the conditions but can account for the pattern of samples observed as well as the relationship between sampling bias and accuracy.

In general terms, our simulations show how, on its own, the opportunity to select information is not necessarily enough to improve performance relative to, for example, observing another person's actions. First, it must be possible for the learner to select information to test his or her specific hypothesis. Second, this must facilitate the adoption of new hypotheses that are more likely to be correct. If either new instances or new hypotheses are generated randomly, an ability to select data will not lead to any advantage.

General Discussion

In their landmark work on concept acquisition, Bruner et al. (1956) discussed the distinction between learning through *selection*, a self-directed information-sampling strategy, and

Table 2
Best Fit Parameters

Symbol	Value	Parameter meaning
n	8	Number of observations
α	0.2	Prior weight
p_θ	0.3	Probability of modifying θ
s_b	0.15	Width of proposal distributions for b
S	0.1	Width of selection distribution

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

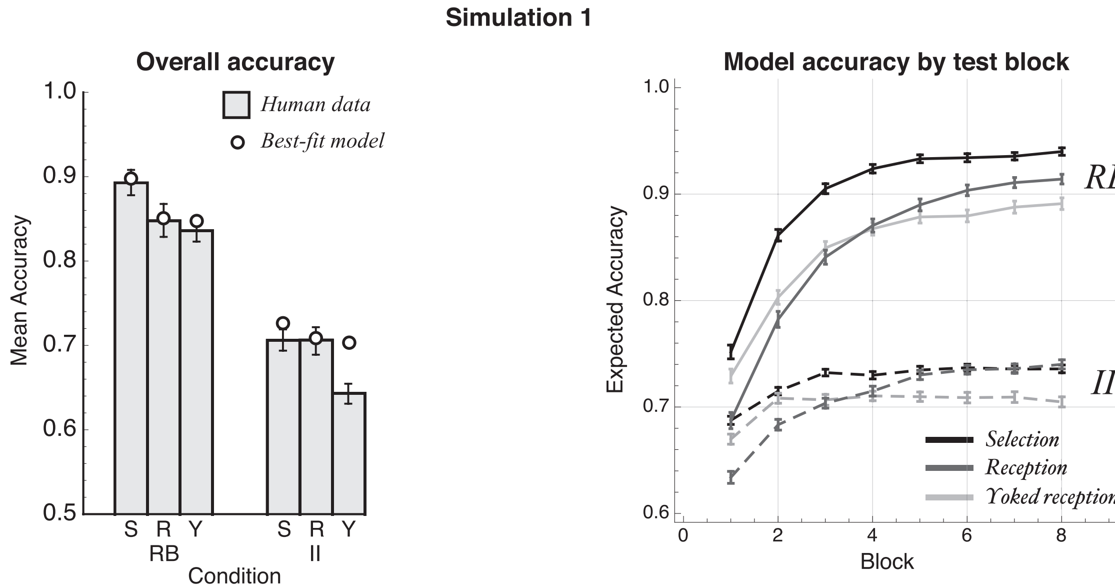

Figure 12. Simulation 1 results. Left: Overall expected accuracy by training condition for human participants (bars) and best fitting model (circles). Right: Accuracy by test block for best fit model. S = Selection; R = Reception; Y = Yoked reception; RB = rule based; II = information integration.

learning through *reception*, a passive mode relying on whatever data happens to be encountered in the environment. Both forms of learning have been the focus of extensive—but relatively independent—research programs in cognitive psychology. Work on hypothesis testing and diagnostic reasoning has sought to understand how people select data to evaluate one or more hypotheses. In contrast, research on concept and category learning has focused predominantly on reception, with the typical experimental approach ensuring that all participants experience the same, carefully controlled distribution of items. One consequence of this focus is that existing models fail to account for how people might make selection decisions during learning, or how these decisions might influence patterns of acquisition (Gigerenzer, 2006). This limits the generalizability of these theories relative to the range of strategies people use during natural category learning.

The aim of the present study was to understand the consequences of selection and reception within the context of a simple, well-studied category learning task. We evaluated two reasons for a selection advantage as compared with reception-based learning. First, selection learning can be more efficient relative to passive reception of data from the environment because it allows the learner to focus on information that will be useful. Second, selection learning can also be superior to "yoked" reception of the same training set because the data that are generated are tied to the sequence of hypotheses held by the selection learner.

We have proposed that, in both cases, this advantage depends on a sampling bias that results from being able to choose what to learn

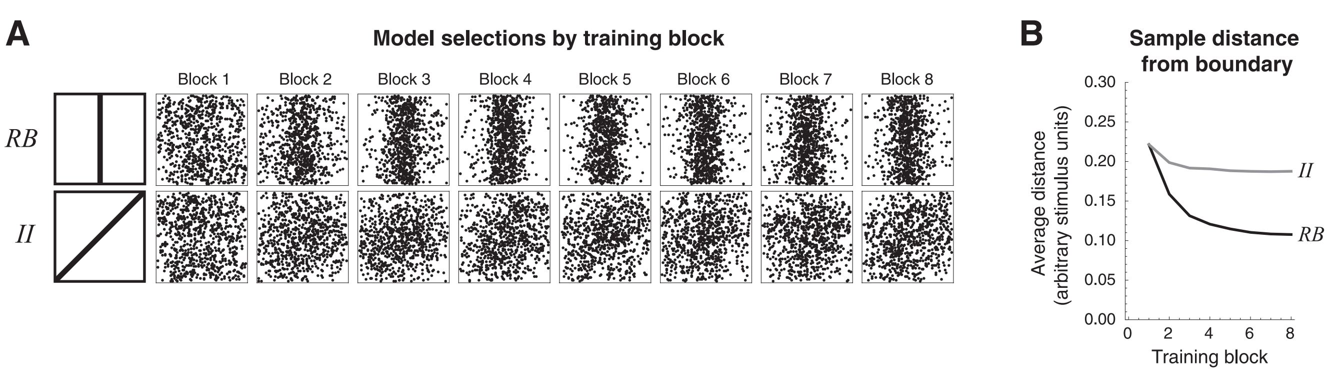

Figure 13. A: Simulated distributions of samples generated by the selection model over the course of training. Similar to the human data in Figure 6, in the rule-based (RB) task, the model's selections are increasingly drawn from the true category boundary over time, whereas in the information-integraion (II) task, they appear more diffuse. B: Average distance of samples from the true category boundary in both tasks from Simulation 1. As in the human data, the average distance is lower in the RB task than in the II task.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

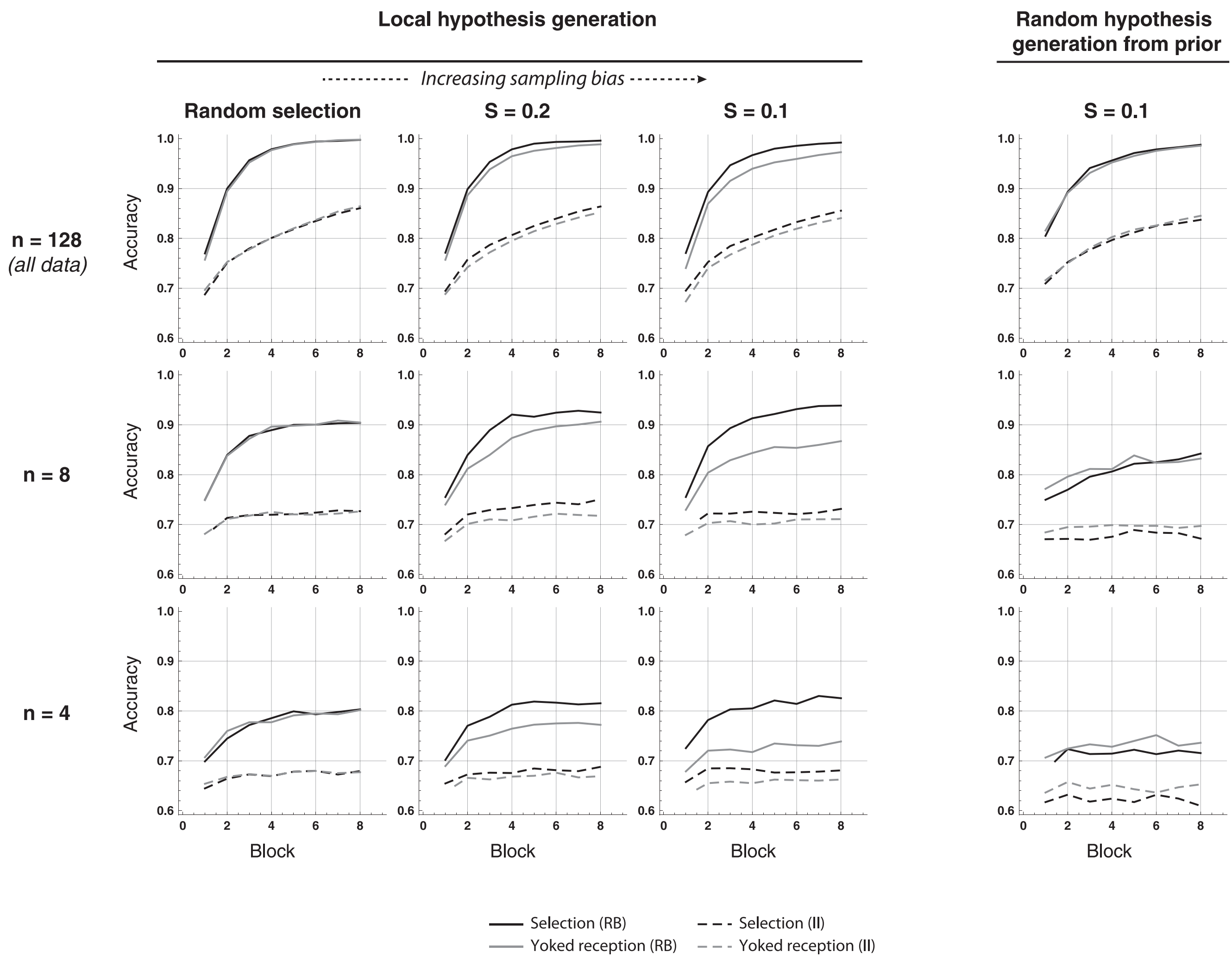

Figure 14. Simulation 2 results, comparing classification accuracy by block for selection (black lines) and yoked reception (gray lines) models in the rule-based (RB) (solid) and information-integration (II) (dashed) tasks. The number of recent observations retained in memory (n) varies across the rows, with the top row indicating the model's performance when all training items are remembered perfectly. The left three columns correspond to the model with a local, stochastic hypothesis-generation process, as described in the text, at three different levels of sampling bias: random, weak ($S = 0.2$) and strong ($S = 0.1$). Under random sampling, there is no difference between selection and yoked reception learners, but as sampling bias increases (i.e., for lower values of S), the two conditions diverge. This divergence is found at all memory capacities but is strongest when few items are stored in memory. The rightmost column provides the same comparison when there is a strong bias, but new hypotheses are generated randomly from the prior on every trial. Under random-hypothesis generation, there is no advantage for selection learning over yoked reception learning, showing that the benefit of making selections depends on their interaction with the hypothesis-generation process.

about. It is important to note that we use the term *bias* in the sense that the selection of new data depends on the learner's current beliefs, rather than to imply that those decisions are incorrect. A selection learner may collect data that are strongly biased by their current hypothesis, but also ineffective for learning, as seen in many kinds of confirmation bias (Nickerson, 1998; see also Denrell & Le Mens, 2011). Accordingly, our goal was not to prescribe either selection or reception as a better learning method in general, but rather to explain the effects of each strategy on a common learning process.

Benefits of Selection Depend on Rule Complexity

Previous research has often attributed a selection learning advantage to being more mentally "active" or engaged in different kinds of cognitive processing (Chi, 2009; Elio & Lin, 1994; Smalley, 1974; Voss et al., 2011). It is self-evident that selection learning requires active decision making about what information to collect and that this involves processes that are not necessary for receptive, observational learning. It is not clear, however, that an advantage for selection learning (when

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

present) is entirely a by-product of those decision-related processes. Moreover, such an explanation is clearly inadequate when an advantage is *not* found even though selection decisions were involved and all that changed was the form of the target concept (e.g., as observed in our II task).

According to our account, advantages from selection learning can result from a stochastic, sequential hypothesis-testing process. First, when the true category structure is consistent with their prior biases, selection learners can adaptively collect information that is more useful than a typical data set specified by the experimenter. Second, the asymmetry in learning outcomes associated with selection and yoked learners derives directly from the way in which participants search the space of hypotheses during learning, and the simple fact that selection learners choose data to test their specific beliefs, whereas the yoked learner cannot. This account assumes identical hypothesis spaces, learning strategies, parameters, and priors for all groups.

An important inspiration for the present study is machine learning research showing that allowing artificial learners to select information about which they are uncertain can improve learning efficiency in a broad range of real-world problems (Castro et al., 2009; Cohn et al., 1992; Dasgupta et al., 2005; Mackay, 1992; Settles, 2009). Our results show that achieving similar improvements in efficiency in human learners depends on the structure of the concept being learned. In our II task, there was no benefit for selection learning over observation of randomly generated instances. More importantly, the II selection group performed considerably worse than the RB selection group, despite sampling the same amount of data under identical conditions. This finding supports previous demonstrations that the benefits of selection strongly depend on the nature and difficulty of the task (Enkvist et al., 2006; Schwartz, 1966).

In our simulations, we accounted for this difference by assuming that learners have a strong prior bias toward simple, unidimensional rules (Ashby et al., 2002, 1999; Gureckis & Love, 2002), which interferes with the discovery of the correct II rule. This is consistent with how many existing models of category learning have attempted to incorporate aspects of rule learning. For example, RULEX (Nosofsky & Palmeri, 1998) assumes a stochastic process of rule generation and a preference for simple unidimensional rules, while allowing for encoding of individual exceptions to the hypothesized rule. Similarly, COVIS (Ashby et al., 2011) proposes that a rule-learning system represents a set of potential rules (generally single-dimensional or conjunctive/disjunctive rules on two dimensions) that vary in their initial weight, but with preference given to simpler, axis-aligned rules.

Beyond the specifics of this category learning task, our results suggest that the advantages for selection learning strongly depend on the structure of the learning problem and how it relates to prior biases the learner brings to the task. In cases in which learners' prior beliefs are inconsistent with the true concept, their choice of which information to test may be systematically biased, and, as a result, learning may be less successful. These principles may hold important lessons for selection-based learning across a wide range of learning problems, particularly those that involve sequential hypothesis testing.

Different Learning Outcomes From the Same Data: Hypothesis-Dependent Sampling Bias or Sampling Assumptions?

An important finding from our experiment was that yoked participants who saw the same observations performed systematically worse than their counterparts in the selection condition. Whereas this is a key prediction of the hypothesis-dependent sampling bias, this systematic divergence in learning is difficult to explain using existing models that update their beliefs through a common process (e.g., Kruschke, 1992; Love, Medin, & Gureckis, 2004; Nosofsky, 1984). For example, the "weak" Bayesian learning discussed in the introduction is unable to predict the systematic differences between the selection and yoked learners. In some of these theories, the results might be partially accommodated by assuming a difference in some general learning parameters or learning process between groups (e.g., memory strength or learning rate). However, such theories would still need modification to explain other aspects of the data such as the systematic patterns shown in Figure 8.

However, the idea that two learners who experience the same data might learn different things is not without precedent in the literature. For example, recent work with Bayesian models of cognition has argued that people adopt different likelihood functions depending on their assumptions about the process that generated observations (Tenenbaum, 1999; Tenenbaum & Griffiths, 2001). These accounts of *sampling assumptions* differ importantly from our proposal of the *hypothesis-dependent sampling bias* and are worthy of consideration.

The basic idea behind sampling assumptions is that it is often possible to make more specific inferences from a set of positive examples if the learner knows how examples were selected (Mitchell, 1997). According to "weak sampling" (which corresponds to the Bayesian model from the introduction), the learner assumes the examples are chosen independently of the true concept. This is often the default assumption made in traditional Bayesian learning models (see Navarro, Dry, & Lee, 2012, and Tenenbaum & Griffiths, 2001, for a discussion). In contrast, "strong sampling" assumes that the generating process itself is informative about the underlying concept. Selection (or learner-driven) learning is often argued to justify a "weak sampling" assumption because the learner should recognize that his or her own selection decisions are independent of the target concept (because they do not yet know it). In contrast, "strong sampling" is justified in settings in which learners are instructed by a knowledgeable teacher (Gweon, Tenenbaum, & Schultz, 2010; Xu & Tenenbaum, 2007a, 2007b). A third type of sampling is "pedagogical sampling," which entails a more complex set of assumptions about how teachers might select examples to be maximally helpful to the learner (Shafto & Goodman, 2008; Shafto et al., 2012).

Xu and Tenenbaum (2007a) explored how children and adults generalized novel words (e.g., *wug*) given only positive examples of the concept in both learner-driven (i.e., selection) and teacher-driven (reception, roughly speaking) conditions. In the learner-driven condition, participants pointed at objects and requested their label or name. In the teacher-driven condition, the experimenter selected examples and provided a label for the object. The study was designed so that the set of examples viewed by both groups was always identical, the only difference being who chose them (either the learner or the teacher). The results showed that both children and adults made more restrictive generalizations of the novel word in the teacher-driven condition compared with the learner-driven condition, suggesting that knowledge about how samples were gathered influenced learning. Xu and Tenenbaum modeled the learner-driven condition as following the weak sampling assumption (Xu & Tenenbaum, 2007a, p. 292), whereas they assumed strong sampling when a teacher provided examples (such as on the first trial of the learner-driven condition, or in all trials of the teacher-driven condition).

Despite the fact that the learner-driven and teacher-driven distinction introduced by Xu and Tenenbaum (2007a) is very similar to Bruner et al.'s (1956) distinction between selection and reception-based hypothesis testing, it is not clear that sampling assumptions are the most appropriate way to interpret the results of the present study. For example, in most of the work on strong versus weak sampling, the emphasis is on learning generative models from positive examples alone (cf. Navarro et al., 2012; Tenenbaum, 1999; Tenenbaum & Griffiths, 2001; Xu & Tenenbaum, 2007b). In this case, strong sampling assumptions can lead to faster learning because the modified likelihood function can pick out more specific concepts with fewer training examples. However, irrespective of sampling assumptions, the location of a dichotomous category distinction can be accurately specified using only a few training items, especially when those examples fall close to the true category boundary. This tends to lessen the influence that sampling assumptions might have on the learning process. In addition, we found no difference in accuracy between yoked learners (i.e., the naïve vs. aware groups) whether or not they were aware of the source of their training data, suggesting that such knowledge did not play a significant role. Finally, it is unclear that any of our conditions could be construed as "teaching" in the same sense as the teacher-led condition in Xu and Tenenbaum (2007a) or as addressed in theories of pedagogical sampling (Shafto & Goodman, 2008; Shafto et al., 2012).

In sum, although the hypothesis-dependent sampling bias and sampling assumptions are both proposals about how learning might change depending on the role the learner has in gathering the data, they may be complementary theories that apply in different learning contexts. In our study, it appears that the relationship between sampling biases and the hypothesis-generation process provides a better account.

The Source of Hypothesis-Dependent Sampling Biases

As shown in Figure 6, our selection procedure provides a rich source of data about how people search for information while learning. Perhaps the most striking pattern was that participants in the RB task concentrated their samples near the true category boundary in the latter part of the task. We have shown that this pattern could arise by assuming that people are more likely to select items near their currently hypothesized category boundary. Over time, as their hypothesis becomes more similar to the true boundary, samples are increasingly clustered close to it as well. Selecting items from the boundary of the current hypothesis is a reasonable strategy if there is greater uncertainty within that region of the stimulus space, in which case those items will convey more useful information for learning. Of course, it is possible that selection learners' ability to sample in this region simply reflects learning that has already occurred. This seems unlikely, however, given that participants continue to sample near the category boundary even after achieving high performance (in the RB task) despite the added effort required to design samples.

The question of how humans make intuitive judgments about the "usefulness" of new observations has been the focus of extensive study in the hypothesis-testing literature (Ginzberg & Sejnowski, 1996; Klayman & Ha, 1987; Oaksford & Chater, 1994; Skov & Sherman, 1986; Wason, 1960). Early findings raised concerns about the human ability to identify the value of future information (e.g., the pervasive evidence of confirmation bias; Wason, 1960; see also Nickerson, 1998, for a review). Our study provides a counterexample to this well-known effect in that participants quite effectively sought discriminating information about the categories in the RB task. However, at this point, our sampling data do not uniquely support any particular model of selection decisions (e.g., impact, information gain, and probability gain all make somewhat similar predictions in the current task; see Nelson, 2005). In our simulations, we simply assumed that those decisions were systematically biased by the learner's current hypothesis. However, Markant and Gureckis (2012) used a more complex category structure to distinguish the information-sampling strategy that selection participants used. In that study, participants were asked to report their uncertainty about how to classify items that they selected, and the authors found that particpants frequently chose items that fell near the subjective boundary between two possible categories (similar to the finding of sampling at the category boundary explored here).

It is interesting that participants in the II task were generally unable to sample as effectively as RB participants. It is undoubtedly more difficult to design samples that fall along the diagonal because it requires specifying values along both stimulus dimensions. Notwithstanding the increased effort required, one explanation might be that participants in the II task *are* in fact sampling effectively (i.e., in regions of the space for which they are most uncertain). However, this uncertainty is evaluated *with respect* to the limited and biased set of hypotheses the learner is actually considering at any point in time (Bonawitz & Griffiths, 2010). Consistent with the above discussion, our decision-bound analyses suggested that learners in the II task persisted in using simple, unidimensional rules throughout the task. Thus, the pattern of diffuse selections in the II task could reflect sampling along a suboptimal and frequently shifting unidimensional boundary. Because II learners in our task never achieved the same high performance as RB participants, we were unable to observe whether the same convergence of samples on the true decision boundary might occur.

Although our account of a hypothesis-dependent sampling bias can account for many aspects of our results, the actual pattern of sampling data we observed in our experiment might reflect a mixture of multiple processes or a shift in strategy over time. For example, selection may serve an adaptive memory function by allowing participants to remind themselves of past experiences that have been forgotten. Indeed, certain instances of what might look like "confirmation bias" in our task (e.g., selecting an item that should be easy to classify given what they have already seen) may simply be participants' attempt to remind themselves about particular regions of the stimulus space or to verify that the task

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

structure has not changed. In general, however, the results are consistent with people selecting items that they are uncertain about how to classify.

Relation to Other Theories of Sequential Hypothesis Testing

Our modeling framework draws heavily from a large and influential literature on sequential hypothesis testing (Gregg & Simon, 1967; Millward & Spoehr, 1973; Nosofsky & Palmeri, 1998; Thomas et al., 2008; Trabasso & Bower, 1964). According to these models, learners maintain a limited set of active hypotheses at any given point during learning and use new observations to eliminate entries in that set. The makeup of the current set of hypotheses influences how one evaluates data and generates new hypotheses (Thomas et al., 2008).

One difference is that our proposal for the mechanism by which new hypotheses are generated draws from Monte Carlo techniques developed in the Bayesian statistics literature (Metropolis & Ulam, 1949). Such approaches have recently gathered considerable attention in the cognitive modeling literature because they help to relate both computational and mechanistic levels of explanation (Brown & Steyvers, 2009; Ullman, Goodman, & Tenenbaum, 2010). It is interesting to consider the relationship between these newer approaches and the classic models of hypothesis testing that are also based on Markov processes. Although sharing many features in common, there are a number of attractive features of the new approach that go beyond older work on serial hypothesis testing. First, so-called rational process models allow one to specify a prior over hypotheses, which is particularly useful when that prior governs an abstract property of many hypotheses (e.g., a preference for simple rules on any one dimension). Second, because the models are approximations to a fully Bayesian solution, they allow characterization of suboptimalities in people's behavior (Brown & Steyvers, 2009). Understanding how people diverge from an optimal solution may help to narrow the range of potential process models that are appropriate.

Conclusions

Both selection and reception are common ways for people to learn in the real world, but they have rarely been compared directly in experimental contexts. Our results join a growing number of studies in demonstrating that, under certain conditions, people can effectively speed their own learning through self-directed exploration of their environment (Atkinson, 1972a, 1972b; Castro et al., 2009; Fazio, Eiser, & Shook, 2004; Sobel & Kushnir, 2006; Steyvers et al., 2003). We found that selection learners were able to make informative queries to support their learning, but were more successful at doing this for RB categories than II categories. In addition, selection learners systematically outperformed participants who were yoked to their observations. We have argued that accounting for the differences between selection and reception depends on a better understanding of how people use their existing hypotheses to collect information, and how the outcome of that process changes the potential for learning relative to passive observation.

References

Ashby, F. G., Alfonso-Reese, L. A., Turken, U., & Waldron, E. M. (1998). A neuropsychological theory of multiple systems in category learning. *Psychological Review, 105,* 442–481.

Ashby, F., Boynton, G., & Lee, W. (1994). Categorization response time with multidimensional stimuli. *Perception & Psychophysics, 55,* 11–27.

Ashby, F. G., Maddox, W. T., & Bohil, C. J. (2002). Observational versus feedback training in rule-based and information-integration category learning. *Memory & Cognition, 30,* 666–677.

Ashby, F., Paul, E., & Maddox, W. (2011). COVIS. In E. Pothos & A. Willis (Eds.), *Formal approaches in categorization* (pp. 65–87). New York, NY: Cambridge University Press.

Ashby, F., Queller, S., & Berretty, P. (1999). On the dominance of unidimensional rules in unsupervised categorization. *Perception & Psychophysics, 61,* 1178–1199.

Atkinson, R. (1972a). Ingredients for a theory of instruction. *American Psychologist, 27,* 921–931.

Atkinson, R. (1972b). Optimizing the learning of a second-language vocabulary. *Journal of Experimental Psychology, 96,* 124–129.

Austerweil, J., & Griffiths, T. (2008). *A rational analysis of confirmation with deterministic hypotheses*. In B. C. Love, K. McRae, & V. M. Sloutsky (Eds.), *Proceedings of the 30th Annual Conference of the Cognitive Science Society* (pp. 1041–1046). Austin, TX: Cognitive Science Society.

Bacon, F. (1902). *Novum organum*. New York, NY: P. F. Collier. (Original work published 1620)

Bonawitz, E., & Griffiths, T. (2010). Deconfounding hypothesis generation and evaluation in Bayesian models. In S. Ohlsson & R. Catrambone (Eds.), *Proceedings of the 32nd Annual Conference of the Cognitive Science Society* (pp. 2260–2265). Austin, TX: Cognitive Science Society.

Brown, S., & Steyvers, M. (2009). Detecting and predicting changes. *Cognitive Psychology, 58,* 49–67.

Bruner, J. (1961). The act of discovery. *Harvard Educational Review, 31,* 21–32.

Bruner, J., Goodnow, J., & Austin, G. (1956). A study of thinking. New York, NY: John Wiley and Sons.

Bruner, J., Jolly, A., & Sylva, K. (1976). Play: Its role in development and evolution. New York, NY: Basic Books.

Carey, S. (1978). The child as word learner. In M. Halle, J. Bresnan, & G. Miller (Eds.), *Linguistic theory and psychological reality* (pp. 264–293). Oxford, United Kingdom: MIT Press.

Castro, R., Kalish, C., Nowak, R., Qian, R., Rogers, T., & Zhu, X. (2009). Human active learning. In D. Koller, D. Schuurmans, Y. Bengio, & L. Bottou (Eds.), *Advances in neural information processing systems* (Vol. 21, pp. 241–249). Retrieved from: http://books.nips.cc/nips21.html

Chi, M. (2009). Active-constructive-interactive: A conceptual framework for differentiating learning activities. *Topics in Cognitive Science, 1,* 73–105.

Cohn, D., Atlas, L., & Ladner, R. (1992). Improving generalization with active learning. *Machine Learning, 15,* 201–221.

Dasgupta, S., Kalai, A., & Monteleoni, C. (2005). Analysis of perceptron-based active learning. In P. Auer & R. Meir (Eds.), *18th Annual Conference on Learning Theory, COLT 2005* (pp. 249–263). Bertinoro, Italy: Springer.

Denrell, J., & Le Mens, G. (2011). Seeking positive experiences can produce illusory correlations. *Cognition, 119,* 313–324.

Elio, R., & Lin, K. (1994). Simulation models of the influence of learning mode and training variance on category learning. *Cognitive Science, 18,* 185–218.

Enkvist, T., Newell, B., Juslin, P., & Olsson, H. (2006). On the role of causal intervention in multiple-cue judgment: Positive and negative effects on learning. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 32,* 163–179.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

Erickson, M., & Kruschke, J. (1998). Rules and exemplars in category learning. *Journal of Experimental Psychology: General, 127,* 107–140.

Fazio, R., Eiser, J., & Shook, N. (2004). Attitude formation through exploration: Valence asymmetries. *Journal of Personality and Social Psychology, 87,* 293–311.

Feldman, J. (2000). Minimization of Boolean complexity in human concept learning. *Nature, 407,* 630–633.

Garner, W., & Felfoldy, G. (1970). Integrality of stimulus dimensions in various types of information processing. *Cognitive Psychology, 1,* 225–241.

Gigerenzer, G. (2006). What's in a sample? A manual for building cognitive theories. In K. Fiedler&, P. Juslin (Eds.), *Information sampling and adaptive cognition* (pp. 239–260). Cambridge, United Kingdom: Cambridge University Press.

Ginzberg, I., & Sejnowski, T. (1996). Dynamics of rule induction by making queries: Transition between strategies. In *Proceedings of the 18th Annual Conference of the Cognitive Science Society* (pp. 121–125). Hillsdale, NJ: Lawrence Erlbaum Associates.

Gregg, L., & Simon, H. (1967). Process models and stochastic theories of simple concept formation. *Journal of Mathematical Psychology, 4,* 246–276.

Gregory, R. (1970). *The intelligent eye*. New York, NY: McGraw-Hill.

Gregory, R. (1974). Choosing a paradigm for perception. In E. Carterett & M. Friedman (Eds.), *Handbook of perception: Historical and philosophical roots of perception* (Vol. 1, pp. 255–283). New York, NY: Academic Press.

Gureckis, T., & Love, B. (2002). Who says models can only do what you tell them? Unsupervised category learning data, fits, and predictions. In *Proceedings of the 24th Annual Conference of the Cognitive Science Society* (pp. 399–404). Hillsdale, NJ: Lawrence Erlbaum Associates.

Gureckis, T., & Markant, D. (2009). Active learning strategies in a spatial concept learning game. In N. Taatgen, H. van Rijn, L. Schomaker, & J. Nerbonne (Eds.), *Proceedings of the 31st Annual Conference of the Cognitive Science Society* (pp. 3145–3150). Austin, TX: Cognitive Science Society.

Gureckis, T. M., & Markant, D. B. (2012). Self-directed learning: A cognitive and computational perspective. *Perspectives on Psychological Science, 7,* 464–481.

Gweon, H., Tenenbaum, J., & Schultz, L. (2010). Infants consider both the sample and the sampling process in inductive generalization. *Proceedings of the National Academy of Sciences, 107, 9066–9071*.

Heller, K., Sanborn, A. N., & Chater, N. (2009). Hierarchical learning of dimensional biases in human categorization. In J. Lafferty & C. Williams (Eds.), *Advances in neural information processing systems* (Vol. 22, pp. 727–736). Retrieved from http://books.nips.cc/nips22.html

Hunt, E. (1965). Selection and reception conditions in grammar and concept learning. *Journal of Verbal Learning and Verbal Behavior, 4,* 211–215.

Huttenlocher, J. (1962). Effects of manipulation of attributes on efficiency of concept formation. *Psychological Reports, 10,* 503–509.

Kemp, C. (2012). Exploring the conceptual universe. *Psychological Review, 119,* 685–722.

Klahr, D., & Nigam, M. (2004). The equivalence of learning paths in early science instruction: Effects of direct instruction and discovery learning. *Psychological Science, 15,* 661–667.

Klayman, J., & Ha, Y. (1987). Confirmation, disconfirmation, and information in hypothesis testing. *Psychological Review, 94,* 211–228.

Kolb, D. (1984). *Experiential learning: Experience as the source of learning and development*. Englewood Cliffs, NJ: Prentice Hall.

Kruschke, J. (1992). ALCOVE: An exemplar-based connectionist model of category learning. *Psychological Review, 99,* 22–44.

Kruschke, J. (1993). Human category learning: Implications for backpropagation models. *Connection Science, 5,* 3–36.

Kuhn, D., & Ho, V. (1980). Self-directed activity and cognitive development. *Journal of Applied Developmental Psychology, 1,* 119–133.

Lagnado, D. A., & Sloman, S. (2004). The advantage of timely intervention. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 30,* 856–876.

Lagnado, D., & Sloman, S. (2006). Time as a guide to cause. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 32,* 451–460.

Laughlin, P. R. (1972). Selection versus reception concept-attainment paradigms for individuals and cooperative pairs. *Journal of Educational Psychology, 63,* 116–122.

Laughlin, P. (1975). Selection versus reception concept-attainment paradigms. *Journal of Experimental Psychology: Human Learning and Memory, 1,* 319–325.

Love, B., Medin, D., & Gureckis, T. (2004). SUSTAIN: A network model of category learning. *Psychological Review, 111,* 309–332.

Mackay, D. (1992). Information-based objective functions for active data selection. *Neural Computation, 4,* 590–604.

Maddox, W. T., & Ashby, F. G. (2004). Dissociating explicit and procedural-learning based systems of perceptual category learning. *Behavioural Processes, 66,* 309–332.

Markant, D., & Gureckis, T. (2012). One piece at a time: Learning complex rules through self-directed sampling. In N. Miyake, D. Peebles, & R. Cooper (Eds.), *Proceedings of the 34th Annual Conference of the Cognitive Science Society* (pp. 725–730). Austin, TX: Cognitive Science Society.

Metropolis, N., & Ulam, S. (1949). The Monte Carlo method. *Journal of the American Statistical Association, 44,* 335–341.

Millward, R., & Spoehr, K. (1973). The direct measurement of hypothesis-sampling strategies. *Cognitive Psychology, 4,* 1–38.

Mitchell, T. (1997). *Machine learning*. New York, NY: McGraw-Hill.

Montessori, M. (1964). *The Montessori method*. New York, NY: Schocken. (Original work published 1912)

Murphy, K. (2001). *Active learning of causal Bayes net structure* (Technical report). Berkeley, California: Department of Computer Science, University of California, Berkeley.

Navarro, D., Dry, M., & Lee, M. (2012). Sampling assumptions in inductive generalization. *Cognitive Science, 36,* 187–223.

Navarro, D., & Perfors, A. (2011). Hypothesis generation, the positive test strategy, and sparse categories. *Psychological Review, 118,* 120–134.

Nelson, J. (2005). Finding useful questions: On Bayesian diagnosticity, probability, impact, and information gain. *Psychological Review, 112,* 979–999.

Nelson, J., McKenzie, C., Cottrell, G., & Sejnowski, T. (2010). Experience matters: Information acquisition optimizes probability gain. *Psychological Science, 21,* 960–969.

Nickerson, R. (1998). Confirmation bias: A ubiquitous phenomenon in many guises. *Review of General Psychology, 2,* 175–220.

Nosofsky, R. M. (1984). Choice, similarity, and the context theory of classification. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 10,* 104–114.

Nosofsky, R. (1989). Further tests of an exemplar-similarity approach to relating identification and categorization. *Perception & Psychophysics, 45,* 279–290.

Nosofsky, R., & Palmeri, T. (1998). A rule-plus-exception model for classifying objects in continuous-dimension spaces. *Psychonomic Bulletin & Review, 5,* 345–369.

Nosofsky, R. M., Palmeri, T. J., & McKinley, S. C. (1994). Rule-plus-exception model of classification learning. *Psychological Review, 101,* 53–79.

Oaksford, M., & Chater, N. (1994). A rational analysis of the selection task as optimal data selection. *Psychological Review, 101,* 608–631.

Papert, S. (1980). *Mindstorms: Children, computers, and powerful ideas*. New York, NY: Basic Books.

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

Pearl, J. (2000). *Causality: Models, reasoning, and inference*. New York, NY: Cambridge University Press.
Piaget, J. (1930). *The child's conception of physical causality*. New York, NY: Harcourt, Brace.
Platt, J. (1964). Strong inference. *Science, 146,* 347–353.
Poletiek, F. (2011). *Hypothesis-testing behavior*. Hove, UK: Psychology Press.
Popper, K. (1959). *The logic of scientific discovery*. London, United Kingdom: Hutchinson. (Original work published 1935)
Schmidhuber, J. (1991). Curious model-building control systems. In *Proceedings of the International Joint Conference on Neural Networks* (pp. 1458–1463). Singapore: IEEE.
Schober, M., & Clark, H. (1989). Understanding by addressees and overhearers. *Cognitive Psychology, 21,* 211–232.
Schwartz, S. H. (1966). Trial-by-trial analysis of processes in simple and disjunctive concept-attainment tasks. *Journal of Experimental Psychology, 72,* 456–465.
Settles, B. (2009). *Active learning literature survey* (Computer Sciences Technical Report 1648). Madison: University of Wisconsin—Madison.
Shafto, P., & Goodman, N. (2008). Teaching games: Statistical sampling assumptions for learning in pedagogical situations. In B. Love, K. McRae, & V. Sloutsky (Eds.), *Proceedings of the 30th Annual Conference of the Cognitive Science Society* (pp. 1632–1637). Austin, TX: Cognitive Science Society.
Shafto, P., Goodman, N., & Frank, M. (2012). Learning from others: The consequences of psychological reasoning for human learning. *Perspectives on Psychological Science, 7,* 341–351.
Shepard, R. (1964). Attention and the metric structure of the stimulus space. *Journal of Mathematical Psychology, 1,* 54–87.
Shepard, R., Hovland, C., & Jenkins, H. (1961). Learning and memorization of classifications. *Psychological Monographs: General and Applied, 75,* 1–42.
Siskind, J. (1996). A computational study of cross-situational techniques for learning word-to-meaning mappings. *Cognition, 61,* 39–61.
Skov, R., & Sherman, S. (1986). Information-gathering processes: Diagnosticity, hypothesis-confirmatory strategies, and perceived hypothesis confirmation. *Journal of Experimental Social Psychology, 22,* 93–121.
Smalley, N. (1974). Modes of extracting information in concept attainment as a function of selection versus reception paradigms. *Journal of Experimental Psychology, 102,* 56–61.
Snyder, M., & Swann, W. (1978). Hypothesis-testing processes in social interaction. *Journal of Personality and Social Psychology, 36,* 1202–1212.
Sobel, D., & Kushnir, T. (2006). The importance of decision making in causal learning from interventions. *Memory & Cognition, 34,* 411–419.
Steffe, L., & Gale, J. (1995). *Constructivism in education*. Hillsdale, NJ: Lawrence Erlbaum Associates.
Steyvers, M., Tenenbaum, J., Wagenmakers, E., & Blum, B. (2003). Inferring causal networks from observations and interventions. *Cognitive Science, 27,* 453–489.
Tenenbaum, J. B. (1999). Bayesian modeling of human concept learning. In M. Kearns, S. Solla, & D. Cohn (Eds.), *Advances in neural information processing systems* (Vol. 11, pp. 59–65). Cambridge, MA: MIT Press.
Tenenbaum, J., & Griffiths, T. (2001). Generalization, similarity, and Bayesian inference. *Behavioral and Brain Sciences, 24,* 629–640.
Thomas, R., Dougherty, M., Sprenger, A., & Harbison, J. (2008). Diagnostic hypothesis generation and human judgment. *Psychological Review, 115,* 155–185.
Tong, S., & Koller, D. (2001). Active learning for structure in Bayesian networks. In *Seventeenth International Joint Conference on Artificial Intelligence* (pp. 863–869). Seattle, WA: AAAI Press.
Trabasso, T., & Bower, G. (1964). Presolution reversal and dimensional shifts in concept identification. *Journal of Experimental Psychology, 67,* 398–399.
Trabasso, T., & Bower, G. (1968). *Attention in learning: Theory and research*. New York, NY: Wiley.
Trope, Y., & Bassok, M. (1982). Confirmatory and diagnosing strategies in social information gathering. *Journal of Personality and Social Psychology, 43,* 22–34.
Trope, Y., & Liberman, A. (1996). Social hypothesis testing: Cognitive and motivational mechanisms. In E. Higgins & A. Kruglanski (Eds.), *Social psychology: Handbook of basic principles* (pp. 239–270). New York, NY: Guilford Press.
Ullman, T., Goodman, N., & Tenenbaum, J. (2010). Theory acquisition as stochastic search. In S. Ohlsson & R. Catrambone (Eds.), *Proceedings of the 32nd Annual Conference of the Cognitive Science Society* (pp. 2840–2845). Austin, TX: Cognitive Science Society.
Voss, J., Gonsalves, B., Federmeier, K., Tranel, D., & Cohen, N. (2011). Hippocampal brain-network coordination during volitional exploratory behavior enhances learning. *Nature Neuroscience, 14,* 115–120.
Vul, E., Goodman, N., Griffiths, T. L., & Tenenbaum, J. (2009). One and done? Optimal decisions from very few samples. In N. Taatgen, H. van Rijn, L. Schomaker, & J. Nerbonne (Eds.), *Proceedings of the 31st Annual Conference of the Cognitive Science Society* (pp. 148–153). Austin, TX: Cognitive Science Society.
Vul, E., Tenenbaum, J., Griffiths, T., Levy, R., Steyvers, M., & McKenzie, C. (2009). Sympoisum: Rational process models. In N. Taatgen, H. van Rijn, L. Schomaker, & J. Nerbonne (Eds.), *Proceedings of the 31st Annual Conference of the Cognitive Science Society* (pp. 45–46). Austin, TX: Cognitive Science Society.
Wason, P. (1960). On the failure to eliminate hypotheses in a conceptual task. *The Quarterly Journal of Experimental Psychology, 12,* 129–140.
Wason, P. (1966). Reasoning. In B. Foss (Ed.), *New horizons in psychology I* (pp. 135–151). Baltimore, MD: Penguin.
Wason, P. (1968). Reasoning about a rule. *Quarterly Journal of Experimental Psychology, 20,* 273–281.
Xu, F., & Tenenbaum, J. (2007a). Sensitivity to sampling in Bayesian word learning. *Developmental Science, 10,* 288–297.
Xu, F., & Tenenbaum, J. (2007b). Word learning as Bayesian inference. *Psychological Review, 114,* 245–272.

(*Appendix follows*)

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

Appendix

Simulating Selection and Yoked Reception in Unidimensional Rule Learning

In the Implications of a Hypothesis-Dependent Sampling Bias for Learning section, we compared selection and yoked reception in a basic unidimensional problem, the details of which are provided here. The learner's goal is to identify the boundary between two categories along a single feature dimension, represented by the criterion $h^* = 0.5$. The set of possible hypotheses H is composed of any value in the range (0, 1), and each hypothesis $h \in H$ deterministically predicts the category label (Category A when $x \leq h$, Category B otherwise).

Falsification From a Single Observation

First, we considered the likelihood of a learner falsifying a random initial hypothesis following a single observation. We compared three conditions: (a) selection learning with a strong hypothesis-dependent sampling bias, (b) selection learning with a weak sampling bias, and (c) yoked reception. In selection learning, new observations are random samples from a normal distribution centered on the current hypothesis with standard deviation S (for the strong condition, $S = .2$; for the weak condition, $S = .8$). Falsification results from any observation in the interval between the current hypothesis and the true criterion (i.e., where the feedback is inconsistent with the current hypothesis). The probability of falsification is thus the integral of the probability density function of the sampling distribution over that interval.

For the yoked condition, the first observation is generated independently of its initial hypothesis (because the item is chosen by the selection learner). For a given pair of hypotheses held by the selection and yoked learners, the probability of falsification for the yoked learner is given by the integral of the sampling distribution (centered on the selection learner's hypothesis) over the interval between the true boundary and the yoked learner's hypothesis. Given that the two learners' hypotheses are independent, the probability of falsification for the yoked learner is then found by averaging over all possible initial hypotheses that could be held by the linked selection learner.

Under these minimal assumptions, there is a benefit to selecting data in terms of the probability of falsification, with the selection learner having a greater likelihood of falsifying their initial hypothesis (see Figure A1). Moreover, the advantage is greater under a strong bias because the observation is more likely to fall in the critical region between the hypothesized and true boundaries. In yoked reception, however, observations are generated independently of the learner's hypothesis, and as a result, the probability of falsification will be lower (on average) than if the learner was able to choose data to test his or her hypothesis.

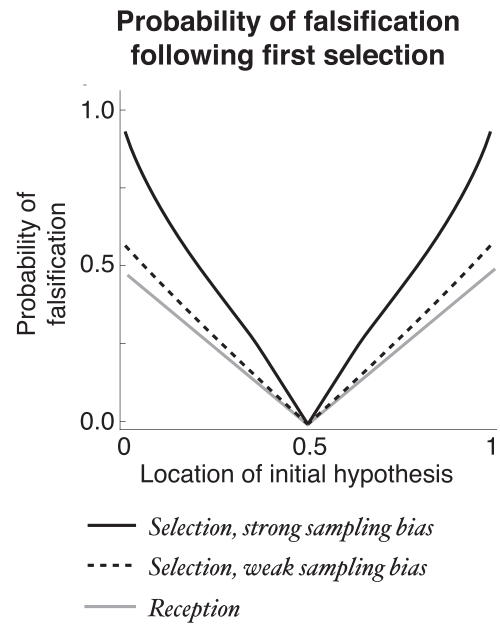

Figure A1. Probability of falsifying a randomly chosen initial hypothesis for unidimensional simulation described in the Appendix. When the initial hypothesis corresponds to the true boundary ($h = 0.5$), the probability of falsifying it is zero for all conditions. For other initial hypotheses, there is an advantage for selection learning over reception. In addition, a stronger bias in the selection condition (i.e., a more narrow distribution centered on the current hypothesis) increases the chances of observing an item that will lead to falsification.

Effect on Learning Efficiency

We next assessed the impact of training condition on learning efficiency over a number of trials. First we compared performance for two learners that begin with the same hypothesis ($h^0 = .1$) but get data through (a) random reception or (b) selection. In the selection condition, new observations were random samples from a normal distribution centered on the current hypothesis (with standard deviation S), as above. In the random-reception condition, new observations were uniformly distributed samples from the full stimulus range. In addition, we measured performance for a separate model that learned through yoked reception of the data generated by the selection model. For the yoked reception model, we focused on the case in which the learner begins with a different hypothesis from that of the selection learner ($h^0 = .9$). We simulated each condition under both weak ($S = .8$) and strong ($S = .2$) hypothesis-dependent sampling bias. Three basic assumptions about the learning process were introduced: (a) The learner only uses the most recent observation to evaluate their hypothesis ($n = 1$),

(*Appendix continues*)

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.

(b) new hypotheses are generated only in response to falsification, and (c) new hypotheses are generated by randomly sampling from H until a consistent hypothesis is found. These assumptions amount to a very simple Win-Stay-Lose-Shift learning rule with "local consistency" (i.e., newly generated hypotheses must be consistent with the observation that led to falsification; see Gregg & Simon, 1967).

We simulated 2,000 runs of the model in each condition, with the observations generated by the selection model used to train the yoked model. Performance was evaluated by measuring the absolute error between the true category boundary ($h^* = .5$) and the model's current hypothesis over the course of 10 training trials. The result (see Figure 4) shows that selection leads to faster learning than both random reception and yoked reception in the context of this simple form of hypothesis testing. Importantly, however, the advantage of selection relative to both reception conditions is attenuated when the sampling bias is weak (see the right panel of Figure 4).

Received September 29, 2011
Revision received January 25, 2013
Accepted January 25, 2013 ■

This document is copyrighted by the American Psychological Association or one of its allied publishers.
This article is intended solely for the personal use of the individual user and is not to be disseminated broadly.